Arizona
Adult Education Standards
for
Mathematics

College and Career Readiness Standards

ARIZONA ADULT EDUCATION
PROFESSIONAL LEARNING SYSTEM

Dear Colleagues,

The Arizona Department of Education – Adult Education Services has a long history of developing and implementing adult education content standards. The first standards were developed in 1999 by multiple content teams comprised of Arizona adult educators, subject matter experts, and State staff. This is the third revision of the Arizona Adult Education Standards, and these standards were intentionally designed to provide an integrated literacy framework by connecting the standards in English Language Arts (ELA), Mathematics, Science, and History and Social Science.

Additionally, Title II: Workforce Innovation and Opportunity Act states, "…agencies who receive Federal adult education funds must align content standards for adult education with State-adopted challenging academic content standards…" Furthermore, "…they must identify curriculum frameworks and align rigorous content standards that specify what adult learners should know and be able to do in the areas of: reading and English Language Arts, mathematics, and English Language Acquisition."

The adult education standards revision process has been a two-year state leadership initiative led by the State office, in collaboration with select adult educators and subject matter experts from around the state. Four content work groups were responsible for revising the content standards in each of their respective disciplines: ELA, mathematics, science, and history and social science. The Adult Education Standards Task Force was responsible for reviewing the standards and draft products developed by the four content work groups and assisting State staff in the standards revision process.

The adult education content standards have been developed and revised to ensure adult students are learning at a high level to prepare them for post-secondary education and training, the workplace, and civic participation. These standards are college and career readiness standards and are intended to be used by adult educators to guide standards-based instruction.

Thank you for all you do for Arizona's adult education students!

Sincerely,
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Tips for Navigating the Electronic Standards Document

The Arizona Adult Education Content Standards document has been designed for adult educators to view and explore electronically. Please see the tips below for navigation assistance.

For educators who prefer to print a copy of the standards, it may be helpful to print specific sections, such as the standard charts.

- Hyperlinks
  - Use the hyperlinks embedded throughout the document to easily navigate to different sections.
    - If you are not seeing hyperlinks, follow the steps below in Word to correct this
      1. File menu → Options → Advanced
      2. Uncheck the box that says Use CTRL + Click to select hyperlinks
      3. Click **OK**
  - Each section in the Table of Contents contains a link to that section of the document.
  - Other links to
    - Supporting documents, such as the Glossary or Appendices
    - outside resources
- To return to your original location after navigation
  - Press **Alt** + ←
  - This process may be repeated if several navigations have occurred.
- Search for a specific term
  - Press **Ctrl** + **F**
  - Enter a word or phrase in the window
  - Click **Enter**

General Introduction to Arizona Adult Education Content Standards

Purpose of the Standards

The purpose of the Arizona Adult Education College and Career Readiness Content Standards is to assist adult educators to better meet the needs of students by providing a framework of what students need to know and be able to do in order to progress through the educational functioning levels (EFLs). Skills barriers, such as in literacy, problem-solving, technology, or workplace employability, often prevent students from reaching their long-term education and career goals. It is essential that adult educators provide students with the opportunity to acquire these skills. The standards are intended to define the knowledge, understanding, and skills needed for adult students to be ready to succeed in post-secondary education and training, without the need for remediation, as well as in the workplace and civic participation.

Background

Based upon requirements in Title II: Workforce Innovation and Opportunity Act (WIOA), Arizona is required to have content standards for adult education that align with high-quality, state-adopted content standards for grades K-12. The content standards in this document accomplish that, having been developed and vetted through the process detailed below. They also reflect academic requirements for success in college-entry, credit-bearing courses, the workplace, and civic participation.

Process for Developing the Standards

In September 2016, the Arizona Department of Education-Adult Education Services opened the standards revision application process to all practicing adult educators and subject matter experts in the state. Applications were vetted and applicants selected, based on their experience and skill sets, to serve on the Standards Task Force and/or Content Work Groups (CWGs). These groups were charged with reviewing, revising, and integrating Arizona's Adult Education College and Career Ready Standards for the following content areas: English Language Arts, Mathematics, History and Social Sciences, and Science. CWGs reviewed research and recommendations from subject matter and standards experts to revise and hone the Arizona adult education standards, ensuring alignment with current Arizona K-12 standards as required by WIOA.

In revising the Arizona adult education standards, the teams were compelled to retain the character of world-class standards (not minimal competencies) customized for adult learners. The revised standards reflect sensible criteria and have been intentionally designed to be useful, intelligible, rigorous, and measurable. The standards focus on academics, contain a balance of skills and content, and represent a reasonable pattern of cumulative learning that is manageable given the time constraints of adult students.

The standards, refined through successive drafts and numerous rounds of feedback, build upon the best elements of standards-related work to date. These standards are intended to be living documents; as new research is validated, they will be revised accordingly.

What the Standards *Are*

Standards are adopted at the state level and guide what students need to know, understand, and be able to do. They define the knowledge and skills in each content area and across domains through a range of cognitive demand levels.

- The Standards are
    - focused in a coherent progression from ABE 1 – ABE 6.
    - research- and evidence-based.
    - rigorous, requiring application of knowledge and demands of higher-level thinking.
    - consistent with post-secondary education and workplace expectations.
    - aligned to the Arizona K-12 standards as required by WIOA.

A standard was included in the document only when the best available evidence indicated that its mastery was essential for college- and career-readiness in a twenty-first-century, globally-competitive society.

What the Standards Are ***Not***

The standards are <u>not</u> ***curriculum***. Unlike the standards, curriculum is adopted at the local program level. While the Arizona adult education standards should be used as the basis for selecting and/or developing a curriculum, they are not a curriculum in and of themselves.

The Arizona Department of Education defines curriculum as:

- the sequence of key concepts, skills, strategies, processes, and assessments that align and support student learning of the standards.
- resources used for teaching and learning the standards.

The standards are <u>not</u> ***instruction***. The standards do not dictate the methods and practices used to effectively teach adult learners (andragogy). Instead, identifying the appropriate method(s) and sequence of instruction at each Educational Functioning Level (EFL)– what will be taught and for how long – requires concerted effort and attention at the program level.

The Arizona Department of Education defines instruction as:

- the methods or methodologies used by teachers to teach their students.
- the techniques or strategies that teachers use in response to the needs of their students.

Standards Implementation

It is essential that adult educators understand that standards are not to be taught in isolation. New learning is about extending knowledge from prior learning to new situations, especially for adult students. For this reason, teachers must understand the progressions in the standards to help students progress from one level to another. Teachers need to understand what individual students already know and where they are heading.

It should be noted that no set of level-specific standards can fully reflect the wide range of abilities, learning goals, learning rates, or achievement levels of students in any given classroom. The Arizona adult education content standards do not define the intervention methods necessary to guide and support students. However, for the standards to be implemented fully, teachers should provide differentiation for students by providing curriculum and instruction at students' appropriate educational levels.

The standards should be implemented so that all students are able to fully participate in their educational programs, including students with disabilities and learning differences. At the same time, all students must have the opportunity to learn and to meet the highest educational functioning levels in the standards to gain access to the knowledge and skills necessary to reach their education, training, and career goals.

Standards-Based Instruction

The Arizona Adult Education Teacher Standards in English Language Arts (ELA), Mathematics, and English Language Acquisition for Adults (ELAA) provide the structure for what *teachers* need to know and be able to do. The teacher standards address standards-based instructional practices, foundational knowledge and skills to effectively teach adult learners, proficiency standards in specific content areas, and professional practices for all adult education teachers. It is imperative that the Arizona teacher standards be used as the foundation to guide teaching and learning at the local program level for Arizona adult educators.

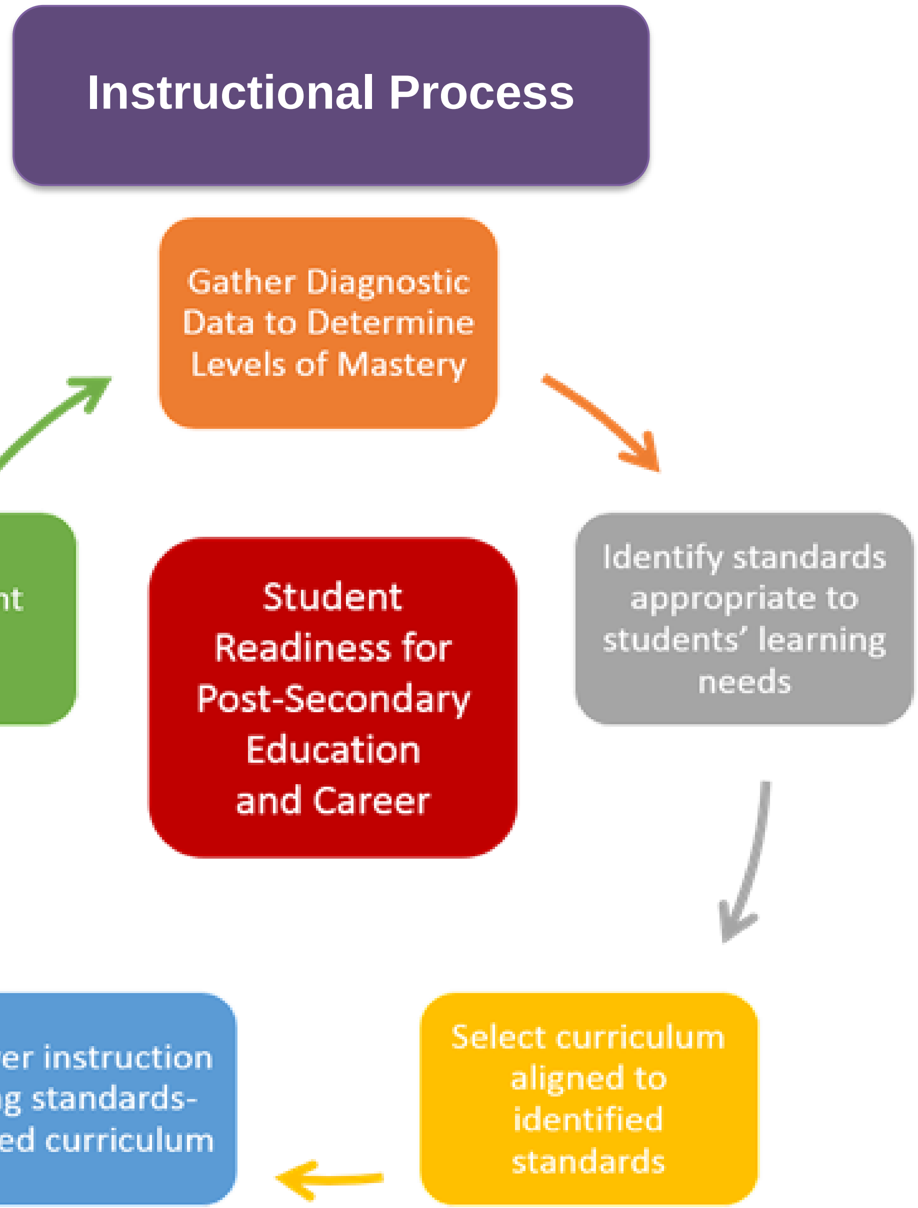
As previously noted, content standards are neither instruction nor curriculum. However, standards must be used to determine which curricular resources, both print and digital, that teachers will use for instruction. In addition, standards guide the scope and sequence of the curriculum to be delivered to students. Diagnostic data is required to determine students' educational levels and their mastery of standards, both upon initial enrollment, as well as throughout their educational programs.

While teachers often use standardized tests to make these determinations, this is not the only student data that teachers should be collecting. In addition, formative assessments (used to make ongoing instructional decisions) such as pretests, reading diagnostic assessments, and student work, should also be used to determine levels of mastery. Because these can be done easily within the classroom on a frequent basis, these formative assessments provide the instructor with much more information about student learning to plan for meaningful and appropriate instruction.

It should be noted that, while the process begins with collecting initial data to plan for instruction, there should be continual monitoring and adjusting of this process. It is often necessary to back up or repeat steps throughout the learning cycle process.

Figure 1

The Instructional Process begins with gathering diagnostic data for each student and is an iterative process. Formative assessments are to be used throughout the instructional process to inform instruction.



Introduction to the Arizona Adult Education Mathematics Standards

Purpose of the Mathematics Standards

The Arizona Adult Education Mathematics (Math) Standards address the important mathematical understandings and competencies necessary to build a solid foundation for all adult students. The need to understand and use a variety of mathematical strategies in multiple contextual, real-world situations has never been greater. Application of mathematics continues to be an integral part of everyday life in the workplace and in credit-bearing college coursework. Mathematics education should enable students to fulfill personal ambitions and career goals in the "Information Age." These standards have been designed to help students successfully meet their goals.

Major Shifts from 2012 Standards

There are three key shifts in the Math Standards: Focus, Coherence, and Rigor. These shifts are integral to teaching mathematics.

1) Focus

The standards focus on what is important for students to know. Focus works in conjunction with *coherence* and *rigor*, and addresses the idea that some concepts are more important than others, requiring more time and attention, leading to deeper understanding.

2) Coherence

Mathematical concepts must be coherent, with clear connections across and within domains and levels. Coherence allows students to demonstrate new understanding built on foundations from prior learning.

3) Rigor

Rigor emphasizes *conceptual understanding*, *procedural skill and fluency*, and *application*—all with equal emphasis, but not necessarily equal time. Each of these is necessary to solve problems inside and outside the math classroom. *(See Figure 2 below.) F*or standards to be considered "rigorous," they must include a balance of standards that describe **concepts that need to be understood** and standards that describe the **application of concepts**.

Conceptual Understanding:

When a student *understands* a mathematical concept, they move fluidly between the concrete and abstract. There is evidence of being able to make sense of and justify mathematical connections. Evidence of understanding includes connections among:

- Verbal or written reasoning
- Pictorial representations
- Real-world application
- Procedures/computation

Fluency

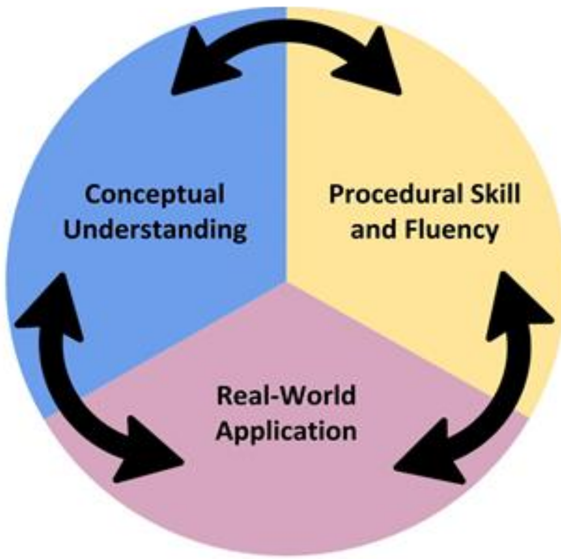
Wherever the words *fluency* or *fluently* appear in the content standards, the word also means *efficiency, accuracy, flexibility, and appropriate application*. Being fluent means that students can choose the methods and strategies to solve contextual and mathematical problems, understand and are able to explain their approaches, and are able to produce accurate answers efficiently.[1]

- Efficiency: carries out easily, keeps track of sub-problems, and makes use of intermediate results to solve the problem
- Accuracy: reliably produces the correct answer
- Flexibility: knows more than one approach, chooses a viable strategy, and uses one procedure to solve and another procedure to double-check solutions
- Appropriate Application: knows when to apply a specific procedure to solve a contextual or mathematical problem

Figure 2

A Balanced Approach to Rigor*

The Math Standards are coherent, focus on deep mathematical content knowledge, and address a balance of rigor, which includes conceptual understanding, application, and procedural skills and fluency.



*Balanced rigor means equal emphasis, not necessarily equal time.

[1] Adapted from Arizona [K-12] Mathematics Standards Introduction (Arizona Department of Education, High Academic Standards for Students, Dec. 2016)

Educational Functioning Levels (EFLs) – A Quick Look

In program year 2016-2017, the National Reporting System (NRS) changed the labels for EFLs from ABE/ASE levels to ABE 1 – 6. Please see the table below for correlations to previous levels. See Appendix A for a complete explanation of each of the six EFLs.

Current EFLs	Previous EFLs	K-12 Grade Equivalencies
ABE 1: Beginning Literacy	Beginning ABE: Beginning Literacy	K – 1.9
ABE 2: Beginning Basic	ABE I: Beginning Basic	2.0 – 3.9
ABE 3: Low Intermediate	ABE II: Low Intermediate	4.0 – 5.9
ABE 4: High Intermediate	ABE III: Middle Intermediate	6.0 – 8.9
ABE 5: Low Adult Secondary	ASE I: High Intermediate	9.0 – 10.9
ABE 6: High Adult Secondary	ASE II: Adult Secondary	11.0 – 12.9

Integration of Literacy Standards in Mathematics

The Arizona Adult Education English Language Arts (ELA) Standards provide an integrated approach to literacy to help guide instruction throughout all disciplines. Therefore, the ELA standards in reading (R), writing (W), and speaking and listening (SL), should be integrated throughout mathematics instruction. By incorporating ELA Standards, and critical thinking in instruction, educators provide students with opportunities to develop literacy through mathematics. The **Standards for Mathematical Practice (MP)** naturally link to the ELA Standards. By engaging in a multitude of critical-thinking experiences students will

- construct viable arguments through proof and reasoning.
- critique the reasoning of others.
- process and apply reasoning from others.
- synthesize ideas and make connections to adjust the original argument.

The goal of using literacy skills in mathematics is to foster a deeper conceptual understanding of the mathematics and provide students with the opportunity to read, write, speak, and listen within a mathematics discourse community.[2]

[2] Adapted from Arizona [K-12] Mathematics Standards Introduction (Arizona Department of Education, High Academic Standards for Students, Dec. 2016)

Digital Literacy

Digital literacy is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning. Electronic technologies, such as calculators and computers, are essential tools for teaching, learning, and doing mathematics. They provide visual images of mathematical ideas and facilitate organizing and analyzing data, and these tools compute efficiently and accurately. Digital literacy skills can support student investigations in every area of mathematics, including geometry, statistics, algebra, measurement, and number. When digital tools are available, students can focus on decision-making, reflecting, reasoning, and problem solving. Digital tools should not be used as replacement for basic conceptual understandings and math intuitions; rather, they can and should be used to foster those understandings and intuitions.[3]

It is the goal in teaching, learning, and assessing mathematical understanding that digital tools are used appropriately and strategically (MP 5). Students should choose a tool, including a calculator, when it is relevant and useful to the problem at hand. It is suggested that calculators in lower EFLs serve as aids in advancing student understanding without replacing other calculation methods. Calculator use can promote the higher-order thinking and reasoning needed for problem solving in our information- and technology based-society. Their use can also assist teachers and students in increasing student understanding of and fluency with arithmetic operations, algorithms, and numerical relationships and enhancing student motivation. Strategic calculator use can aid students in recognizing and extending numeric, algebraic, and geometric patterns and relationships.[4] It is suggested that teaching, learning, and assessment of mathematics include calculator use on assessments or parts of assessments.[5]

Image credit: www.securedgenetwork.com

[3] National Council of Teachers of Mathematics. (2000). Principles and Standards for School Mathematics. Reston, VA.

[4] National Council of Teachers of Mathematics. (2015). Calculator Use in Elementary Grades-NCTM position statement.

[5] Adapted from Arizona [K-12] Mathematics Standards Introduction (Arizona Department of Education, High Academic Standards for Students, Dec. 2016)

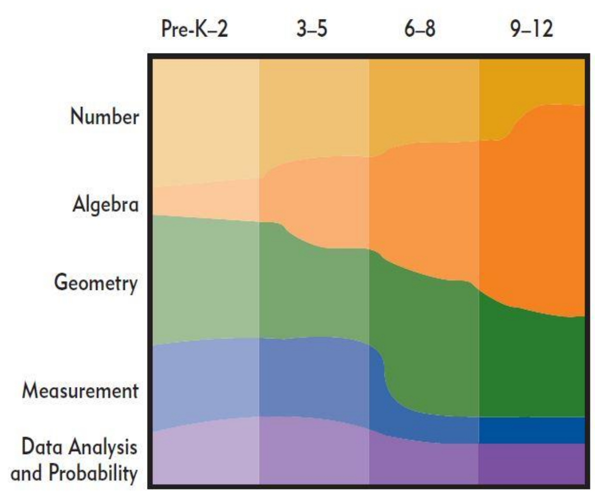
Overview of Math Standards

1. **Mathematical Content Standards**
    - Descriptions of mathematical content that students need to know, understand, and be able to do at each educational functioning level
2. **Standards for Mathematical Practice** (identical for each educational functioning level)
    - "Habits of mind" that students should develop to foster mathematical understanding

The Math Standards have been intentionally designed so mathematical practices and mathematical content are inextricably linked. Together these standards address both "habits of mind" students should develop to foster mathematical understanding, and what students need to know, understand, and be able to do regarding mathematics content. One cannot solve problems without understanding and using mathematical content.[6] These connections are essential in supporting the development of students' broader mathematical understanding. Students who lack understanding of a concept may rely too heavily on procedures and may have difficulty applying the math skills outside of the classroom.

Figure 3a

Principles & Standards School Mathematics

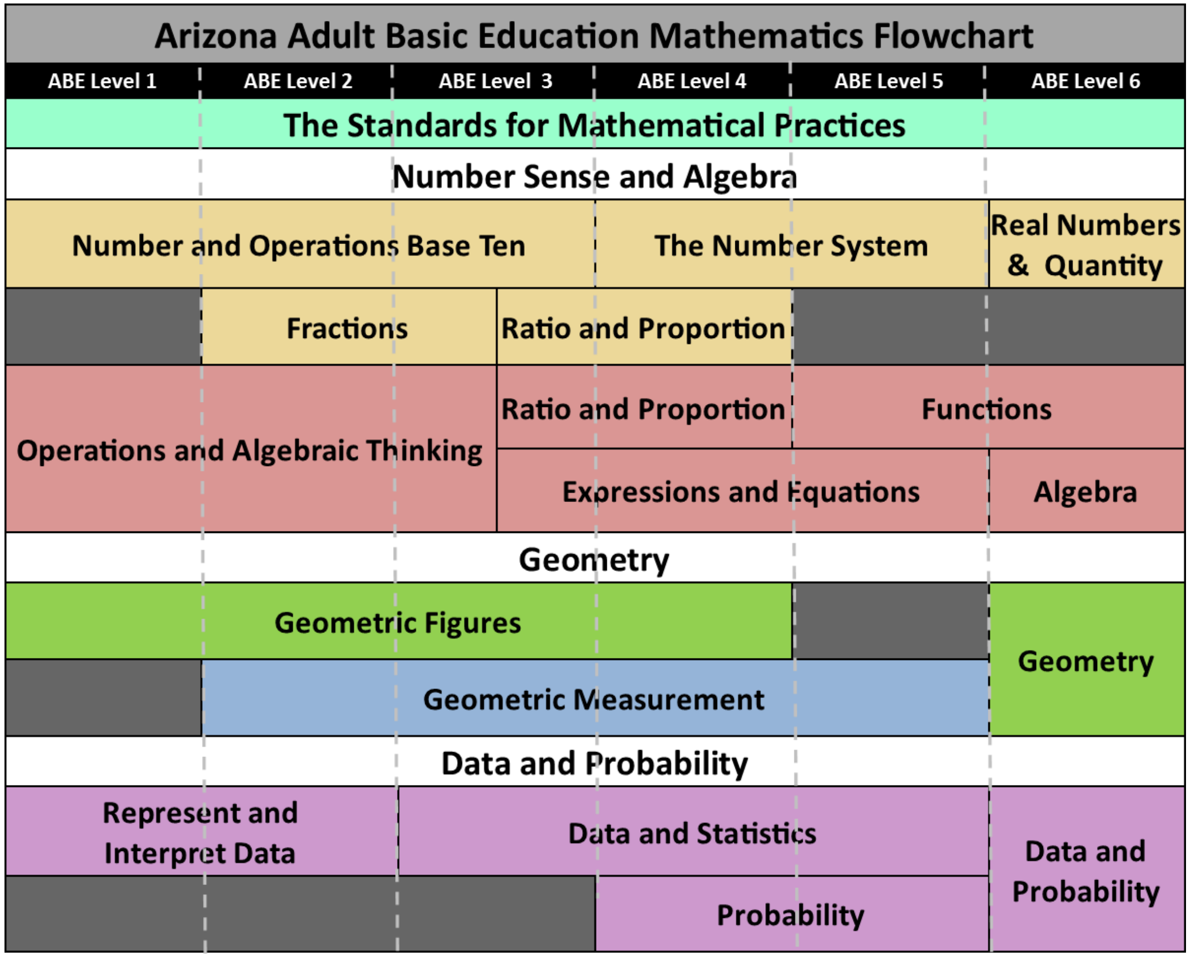


The Content Standards should receive different emphases across the grade bands.

National Council of Teachers of Mathematics, 2000

6 California State Standards-Mathematics (2013)

Figure 3b


Arizona Adult Basic Education Mathematics Flowchart
ABE Level 1
ABE Level 2
ABE Level 3
ABE Level 4
ABE Level 5
ABE Level 6
The Standards for Mathematical Practices
Number Sense and Algebra
Number and Operations Base Ten
The Number System
Real Numbers & Quantity
Fractions
Ratio and Proportion
Operations and Algebraic Thinking
Ratio and Proportion
Functions
Expressions and Equations
Algebra
Geometry
Geometric Figures
Geometry
Geometric Measurement
Data and Probability
Represent and Interpret Data
Data and Statistics
Data and Probability
Probability

The table below contains the four mathematical domains with the clusters arranged in a vertical progression from one EFL to the next EFL. ***This table does* not *replace the standards.*** This 'at a glance' overview builds on the information provided in Figures 3a and 3b.

Figure 3c

	Number Sense and Operations	Algebraic Thinking	Geometry and Measurement	Data Analysis and Statistics
ABE 1	• Number Base 10	• Operations and Algebraic Thinking	• Geometric Figures	• Data and Statistics
ABE 2	• Number Base 10 • Fractions	• Operations and Algebraic Thinking	• Geometric Figures • Geometric Measurement	• Data and Statistics
ABE 3	• Number Base 10 • Fractions	• Operations and Algebraic Thinking • Ratios and Proportions • Expressions and Equations	• Geometric Figures • Geometric Measurement	• Data and Statistics
ABE 4	• Number System • Numerical Expressions	• Ratios and Proportions • Expressions and Equations • Functions	• Geometric Figures • Geometric Measurement	• Data and Statistics • Probability
ABE 5	• Number System • Numerical Expressions	• Ratios and Proportions • Expressions and Equations • Functions	• Geometric Figures • Geometric Measurement	• Data and Statistics • Probability
ABE 6	• Real Number System • Quantities	• Seeing structure in expressions • Arithmetic with Polynomial Expressions • Creating Equations • Reasoning with Equations and Inequalities • Interpreting Functions • Building Functions • Linear, Quadratic, and Exponential Models	• Congruence • Similarity, Right Triangles, and Trigonometry • Expressing Geometric Properties with Equations • Geometric Measurement and Dimension • Modeling with Geometry	• Interpreting Categorical and Quantitative Data • Making Inferences and Justifying Conclusions

Standards for Mathematical Practice

The eight Standards for Mathematical Practice (MP) describe how students are expected to engage within the mathematical content standards. These standards reflect the interaction of skills necessary for success in math coursework, as well as the ability to apply math knowledge and processes within real-world contexts. The Standards of Mathematical Practice highlight the applied nature of math in the workplace and clarify expectations for using mathematics in and outside of the classroom. These standards are intended to complement the mathematics content standards so students increasingly engage with the subject matter as they progress through the educational functioning levels.

Adult educators should seek to develop student expertise through the Standards for Mathematical Practice. Although students exhibit these habits of mind at every level, demonstration of these practices will build, in complexity, through the educational functioning levels. The practices rest on two sets of important processes and proficiencies, the National Council of Teachers of Mathematics (NCTM) Process Standards (1989, 2000), and the Strands of Mathematical Proficiency, as specified in a report by the National Research Council.

Coding for Standards Integration

Where appropriate, the Mathematical Practices are tagged with related standards from the International Society for Technology in Education (ISTE) and the Employability Skills Framework (ESF).

- **International Society for Technology in Education (ISTE)**
  - The math practices that readily appear to lend themselves to digital literacy have been tagged with the appropriate ISTE standard.
    - Not all math practices align with a technology standard.
  - Please click **HERE** to view and/or download the ISTE standards.

- **Employability Skills Framework (ESF)**
  - The math practices that readily appear to lend themselves to the employability skills have been tagged.
    - Not all math practices align with an ESF.
  - Please click **HERE** to view/download the ESF.

The Eight Standards for Mathematical Practices

Figure 4

Overarching Habits of Mathematical Thinkers		
MP1: Make sense of problems and persevere in solving them. **MP6:** Attend to precision.		
Reasoning and Explaining	Modeling and Using Tools	Seeing Structure and Generalizing
MP2: Reason abstractly and quantitatively. **MP3:** Construct viable arguments and critique the reasoning of others.	**MP4:** Model with mathematics. **MP5:** Use appropriate tools strategically.	**MP7:** Look for and make use of structure. **MP8:** Look for and express regularity in repeated reasoning.

MP1: Make sense of problems and persevere in solving them.

Mathematically proficient students

- start by explaining to themselves the meaning of a problem and look for entry points to begin work on the problem.
- plan and choose a solution pathway rather than simply jumping into a solution attempt.
- consider similar problems and try simpler forms of the original problem to gain insight to its solution.
- monitor and evaluate their progress and change course as necessary.
    - For example, they might transform algebraic expressions or change the viewing window on their graphing calculators to get the information they need.
- can explain correspondences between
    - equations,
    - verbal descriptions,
    - tables, and
    - graphs.
- draw diagrams of key features and relationships.
- graph data and search for regularity or trends.
    - For example, less experienced students might rely on using concrete objects or pictures to help conceptualize and solve a problem.

- engage in productive struggle, and continually ask themselves, “Does this make sense?”
- check their answers to problem using different methods.
- can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

[ISTE.1d, ISTE.4d, ISTE.5b, ISTE.5c]

MP2: Reason abstractly and quantitatively.

Mathematically proficient students

- make sense of quantities and their relationships in problem situations.
- can contextualize and decontextualize problems involving quantitative relationships.
- contextualize quantities, operations, and expressions by describing a corresponding situation.
- decontextualize a situation by representing it symbolically.
    - For example, as students manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent.
- know and flexibly use different properties of
    - operations,
    - numbers, and
    - geometric objects.
- when appropriate, they interpret their solutions in terms of the context.

MP3: Construct viable arguments and critique the reasoning of others.

Mathematically proficient students

- construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using
    - concrete,
    - pictorial,
    - or symbolic referents.
        - Arguments may also rely on definitions, assumptions, previously established results, properties, or structures.
- make conjectures and build logical progressions of statements to explore the truth of their conjectures.
    - For example, students can analyze situations by breaking them into cases, and recognize and use counterexamples.
- present their arguments in the form of
    - representations,
    - actions on those representations,
    - and explanations in words (oral or written).
    - critique others by affirming and questioning the reasoning of others.
        - They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it.

- communicate their arguments and compare them to others, and reconsider their own arguments in response to the critiques of others.

[ISTE.1c, ISTE.5a, ISTE.5b, ISTE.6c] [ESF.CS,IS, CTS]

MP4: Model with mathematics.

Mathematically proficient students

- can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
    - This might be as simple as writing an addition equation to describe a situation to:
        - apply proportional reasoning to plan a school event or analyze a problem in the community.
        - use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- can apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that it may need revision later.
- identify important quantities in a practical situation and map their relationships using such tools as,
    - diagrams,
    - two-way tables,
    - graphs,
    - flowcharts,
    - formulas.
        - For example, students can analyze those relationships mathematically to draw conclusions.
- routinely interpret their mathematical results in the context of the situation, and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

[ISTE.1c, ISTE.1d, ISTE.5a, ISTE.5b, ISTE.5c, ISTE.6c] [ESF.CTS]

MP5: Use appropriate tools strategically.

Mathematically proficient students

- consider the available tools when solving a mathematical problem. These tools might include:
    - pencil and paper,
    - concrete models,
    - rulers,
    - protractors,
    - calculators,
    - spreadsheets,
    - computer algebra systems,
    - statistical packages, and
    - dynamic geometry software.

- are sufficiently familiar with tools appropriate for the task at hand to make sound decisions on when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
  - For example, mathematically proficient students analyze graphs of functions and solutions generated using a graphing calculator.
- detect possible errors by strategically using estimation and other mathematical knowledge.
- know that technology can enable them to
  - visualize the results of varying assumptions when making mathematical models,
  - explore consequences,
  - and compare predictions with data.
- identify relevant external mathematical resources, such as digital content located on a website, and use the content to pose or solve problems.
- use digital tools to explore and deepen their understanding of concepts.

[ISTE.1c, ISTE.1d, ISTE.5a, ISTE.5b, ISTE.6c] [ESF.CTS]

MP6: Attend to precision.

Mathematically proficient students

- communicate precisely to others using appropriate mathematical terminology, crafting explanations to convey their reasoning.
- use clear definitions in discussion with others and in their own reasoning.
- state the meanings of the symbols they choose, including using the equal sign, consistently and appropriately.
- understand that "equal" does not mean "the answer is." (For explanation of this concept, see the publication found HERE.)
- understand meanings of symbols used in mathematics.
- calculate accurately and efficiently.
- label quantities appropriately.
- record their work clearly and concisely.
  - For example, less experienced students give carefully formulated explanations to each other. By the time they reach ABE 5, students have learned to examine claims and make explicit use of definitions.

MP7: Look for and make use of structure.

Mathematically proficient students

- look closely to discern a pattern or structure.
  - For example, students might notice that three and seven more is the same amount as seven and three more.
  - For example, students may sort a collection of shapes according to how many sides the shapes have.
- prepare for learning about the distributive property.
  - For example, students understand that 7×8 equals $7 \times 5 + 7 \times 3$.
  - For example, in the expression $x^2 + 9x + 14$, students can see the 14 as 2×7 and the 9 as $2 + 7$.
- recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.
- manage their own progress, stepping back for an overview and shifting perspective when needed.

- can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.
  - For example, the student can see $5 - 3(x - y)^2$ as ‘5 minus a positive number times a square’ and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y.*

  [ISTE.5c]

MP8: Look for and express regularity in repeated reasoning.

Mathematically proficient students

- look for and describe regularities as they solve multiple related problems.
  - For example, students might notice when dividing 25 by 11 that they are repeating the same calculations repeatedly and conclude they have a repeating decimal.
  - Or, by paying attention to the calculation of slope, students repeatedly check whether points are on the line through (1, 2) with slope 3, and can abstract the equation $(y - 2) / (x - 1) = 3$.
    - Noticing the regularity in the way terms cancel when expanding $(x–1)(x+1)$, $(x–1)(x^2+x+1)$, and $(x–1)(x^3 +x^2+x+1)$ might lead them to the general formula for the sum of a geometric series.
- maintain oversight of the process as they work to solve a problem while also attending to the details.
  - continually evaluate the reasonableness of their intermediate results to inform and strengthen their understanding of the structure of mathematics which leads to fluency.

Reading the Math Standards

Standards describe what students should know, understand, and be able to do. They are organized under their appropriate domains into smaller clusters of standards that are related by specific content (see Figure 5). Most clusters have multiple standards associated with them. Standards are numbered. The code for each standard begins with the Educational Functioning Level (ABE 1 – 6), followed by the domain abbreviation, and then the number of the standard (see Figure 6).

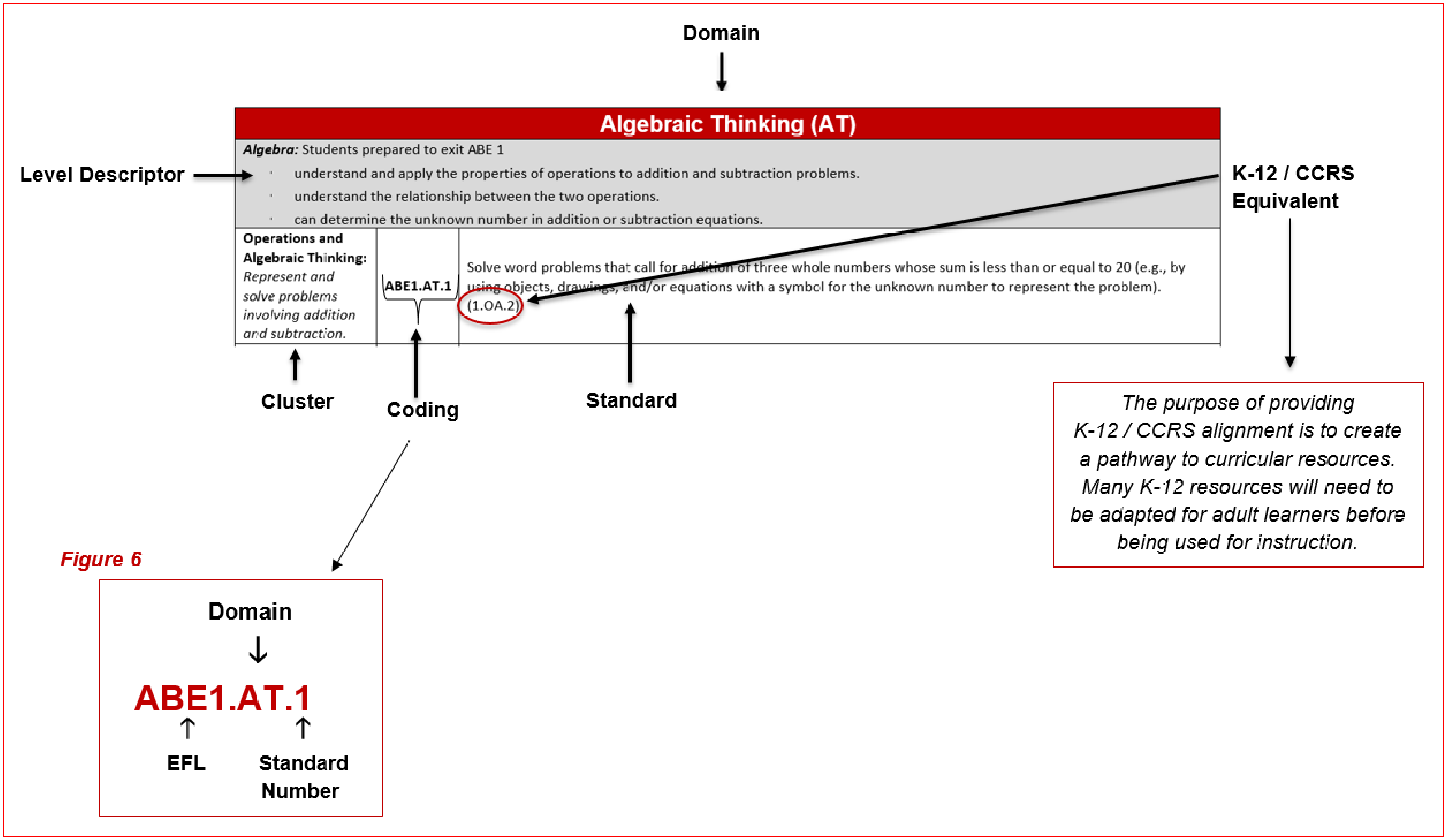


Figure 6

The Mathematics Standards

ABE 1: Beginning Literacy

The Mathematical Practices***:*** Students prepared to exit ABE 1

- can decipher a simple problem presented in a context.
- reason about and apply correct units to the results.
- visualize a situation using manipulatives or drawings.
- explain their processes and results using mathematical terms and symbols appropriate for the level.
- recognize errors in the work and reasoning of others.
- are able to strategically select and use appropriate tools to aid in their work, such as pencil/paper, measuring devices, and/or manipulatives.
- can see patterns and structure in sets of numbers and geometric shapes and use those insights to work more efficiently.

<table>
<tr><th colspan="3">Number Sense and Operations (NO)</th></tr>
<tr><td colspan="3">Number Sense: Students prepared to exit ABE 1
· understand whole number place value for tens and ones.
· can use their understanding of place value to compare two-digit numbers.
· can add whole numbers within 100 and explain their reasoning (e.g., using concrete models or drawings and strategies based on place value and/or properties of operations).
· can apply their knowledge of whole number addition and subtraction to represent and solve word problems that call for addition of three whole numbers whose sum is less than 20 by using such problem-solving tools as objects, drawings, and/or simple equations.</td></tr>
<tr><td>Number Base Ten: Extend the counting sequence.</td><td>ABE1.NO.1</td><td>Count to 120 by 1's, 2's, and 10's starting at any number less than 100. In this range, read and write numerals and represent a number of objects with a written numeral. (1.NBT.1)</td></tr>
<tr><td rowspan="2">Number Base Ten: Understand place value.</td><td>ABE1.NO.2</td><td>Understand that the two digits of a two-digit number represent groups of tens and ones. Understand the following as special cases:
a. 10 can be thought of as a group of ten ones — called a “ten.”
b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). (1.NBT.2)</td></tr>
<tr><td>ABE1.NO.3</td><td>Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. (1.NBT.3)</td></tr>
</table>

Number Base Ten: *Use place value understanding and properties of operations to add and subtract.*	**ABE1.NO.4**	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. (1.NBT.4)
	ABE1.NO.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. (1.NBT.5)
	ABE1.NO.6	Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (1.NBT.6)

Algebraic Thinking (AT)

Algebra: Students prepared to exit ABE 1

- understand and apply the properties of operations to addition and subtraction problems.
- understand the relationship between the two operations.
- can determine the unknown number in addition or subtraction equations.

Operations and Algebraic Thinking: *Represent and solve problems involving addition and subtraction.*	**ABE1.AT.1**	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 (e.g., by using objects, drawings, and/or equations with a symbol for the unknown number to represent the problem). (1.OA.2)
Operations and Algebraic Thinking: *Understand and apply properties of operations and the relationship between addition and subtraction.*	**ABE1.AT.2**	Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.) (1.OA.3)
	ABE1.AT.3	Understands the inverse relationship between addition and subtraction and uses it to find the unknown-addend in a problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8. (1.OA.4)

Operations and Algebraic Thinking: *Add and subtract within 10.*	**ABE1.AT.4**	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). (1.OA.5)
	ABE1.AT.5	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). (1.OA.6)
Operations and Algebraic Thinking: *Work with addition and subtraction equations.*	**ABE1.AT.6**	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false (e.g., Which of the following equations are true and which are false? 6 + 1 = 6 - 1, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2). (1.OA.7)
	ABE1.AT.7	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers (e.g., determine the unknown number that makes the equation true in each of the equations 8 + a = 11, 5 = a - 3, 6 + 6 = a). (1.OA.8)

Geometry and Measurement (GM)

Geometry: Students prepared to exit ABE 1

- can analyze and compare 2- dimensional and 3-dimensional shapes based on their attributes, such as
  - shape,
  - size,
  - orientation,
  - number of sides and/or vertices (angles), and
  - lengths of their sides.
- can reason with two-dimensional shapes (e.g., quadrilaterals and half- and quarter-circles) and with three-dimensional shapes (e.g., right prisms, cones, and cylinders) to create composite shapes.
- can measure the length of an object as a whole number of units, which are not necessarily standard units, (e.g., measuring the length of a pencil using a paper clip as the length unit).

Geometric Figures: *Analyze, compare, create, and compose shapes.*	**ABE1.GM.1**	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). (K.G.4)
Geometric Figures: *Reason with shapes and their attributes.*	**ABE1.GM.2**	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (1.G.2)

Data Analysis and Statistics (DS)

***Data and Probability*: Students prepared to exit ABE 1**

- can organize, represent, and interpret simple data sets (e.g., lists of numbers, shapes, or items) using up to three categories.
- can answer basic questions related to the total number of data points in a set and the number of data points in each category.
- can compare the number of data points in the different categories.

Data and Statistics: *Represent and interpret data.*	**ABE1.DS.1**	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. (1.MD.4)

ABE 2: Beginning Basic

The Mathematical Practices: Students prepared to exit ABE 2

- are able to decipher two-step problems presented in a context, visualizing a situation using diagrams or sketches, and using reasoning and application of the correct units and proper degree of precision to the results.
- can explain their processes and results using mathematical terms and symbols appropriate for the level
- recognize errors in the reasoning of others.
- strategically select and use the appropriate tools to aid in their work, such as pencil/paper, measuring devices, manipulatives, and/or calculators.
- are able to see patterns and structure in sets of numbers, including in multiplication or addition tables, and use those insights to work more efficiently.

Number Sense and Operations (NO)

***Number Sense*:** Students prepared to exit ABE 2

- understand place value for whole numbers to 1000.
- can use that understanding to read, write, count, compare, and round three-digit whole numbers to the nearest 10 or 100.
- are able to compute fluently with all four operations with whole numbers within 100.
- use place value and properties of operations to explain why addition and subtraction strategies work.
- can demonstrate an understanding of the inverse relationship between multiplication and division.
- can solve one- and two-step word problems involving all four operations within 100 and identify and explain arithmetic patterns.
- have an understanding of fractions, especially unit fractions, and can represent simple fractions on a number line.
- understand and can explain equivalence of fractions.
- can recognize and generate simple equivalent fractions.
- can compare two fractions with the same numerator or denominator by reasoning about their size.

Number Base Ten: *Understand place value.*	**ABE2.NO.1**	Understand that the three digits of a three-digit number represent groups of hundreds, tens, and ones (e.g., 706 equals 7 hundreds, 0 tens, and 6 ones and also equals 70 tens and 6 ones). Understand the following as special cases: a. 100 can be thought of as a group of ten tens—called a “hundred.” B. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). (2,NBT.1)
	ABE2.NO.2	Read and write numbers up to 1000 using base-ten numerals, number names, and expanded form. (2.NBT.3)
	ABE2.NO.3	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. (2.NBT.4)

Number Base Ten: *Use place value understanding and properties of operations to add and subtract.*	**ABE2.NO.4**	Fluently add and subtract up to four two-digit numbers using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (2.NBT.5 AND 2.NBT.6)
	ABE2.NO.5	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three- digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. (2.NBT.7)
	ABE2.NO.6	Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. (2.NBT.8)
	ABE2.NO.7	Explain why addition and subtraction strategies work, using place value and the properties of operations. [NOTE: Explanations may be supported by drawings or objects.] (2.NBT.9)
Number Base Ten: *Use place value understanding and properties of operations to perform multi-digit arithmetic.*	**ABE2.NO.8**	Use place value understanding to round whole numbers to the nearest 10 or 100. (3.NBT.1)
	ABE2.NO.9	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (3.NBT.2)
	ABE2.NO.10	Multiply one-digit whole numbers by multiples of 10 in the range 10 to 90 using strategies based on place value and the properties of operations (e.g., $9 \cdot 80$, $5 \cdot 60$). (3.NBT.3)
Fractions: *Understand fractions as numbers.* [NOTE: Expectations for this level are limited to fractions with denominators: 2, 3, 4, 6, 8]	**ABE.2.NO.11**	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. (3.NF.1)
	ABE.2.NO.12	Understand a fraction as a number on the number line; represent fractions on a number line diagram. (3.NF.2)
	ABE.2.NO.13	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (3.NF.3)
	ABE.2.NO.14	Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model). (3.NF.3b)
	ABE.2.NO.15	Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. Examples: express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram. (3.NF.3c)
	ABE.2.NO.16	Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model). (3.NF.3d)

Algebraic Thinking (AT)

Algebra: Students prepared to exit ABE 2

- apply the properties of operations to multiplication and division of whole numbers.
- understand the relationship between multiplication and division.
- can determine the unknown number in multiplication or division equations.

Operations and Algebraic Thinking: *Represent and solve problems involving addition and subtraction.* [NOTE: A range of algorithms may be used.]	**ABE2.AT.1**	Use addition and subtraction within 100 to solve one-step word problems. Use addition to solve two-step word problems using single-digit addends. Represent a word problem as an equation with a symbol for the unknown. [See Table 1.] (2.OA.1)
Operations and Algebraic Thinking: *Add and subtract within 20.*	**ABE2.AT.2**	Fluently add and subtract within 20 using mental strategies. Know from memory all sums of two one-digit numbers. (2.OA.2)
Operations and Algebraic Thinking: *Represent and solve problems involving whole number multiplication and division.*	**ABE2.AT.3**	Interpret products of whole numbers (e.g., interpret 5 • 7 as the total number of objects in 5 groups of 7 objects each). For example, describe a context in which a total number of objects can be expressed as "5 • 7." (3.OA.1)
	ABE2.AT.4	Interpret whole number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each group when 56 objects are partitioned equally into 8 groups, or as a number of groups when 56 objects are partitioned into equal groups of 8 objects each). (3.OA.2)
	ABE2.AT.5	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem). (3.OA.3)
	ABE2.AT.6	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 • a = 48, 5 = a ÷ 3, 6 • 6 = a. (3.OA.4)

Operations and Algebraic Thinking: *Understand properties of multiplication and the relationship between multiplication and division.*	**ABE2.AT.7**	Apply properties of operations as strategies to multiply and divide. Examples: a. If 6 • 4 = 24 is known, then 4 • 6 = 24 is also known. (Commutative property of multiplication.) b. 3 • 5 • 2 can be found by 3 • 5 = 15, then 15 • 2 = 30, or by 5 • 2 = 10, then 3 • 10 = 30. (Associative property of multiplication.) c. Knowing that 8 • 5 = 40 and 8 • 2 = 16, one can find 8 • 7 as 8 • (5 + 2) = (8 • 5) + (8 • 2) = 40 + 16 = 56. (Distributive property.) (3.OA.5)
Operations and Algebraic Thinking: *Multiply and divide within 100.*	**ABE2.AT.8**	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 • 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. Know from memory all products of two one-digit numbers. (3.OA.7)
Operations and Algebraic Thinking: *Solve problems involving the four operations, and identify and explain patterns in arithmetic.*	**ABE2.AT.9**	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Use the Order of Operations when there are no parentheses. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (3.OA.8)
	ABE2.AT.10	Identify patterns in the addition table and the multiplication table and explain them using properties of operations (e.g., observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends). (3.OA.9)

Geometry and Measurement (GM)

***Geometry*:** Students prepared to exit ABE 2

- are able to reason about geometric shapes and their attributes.
- can demonstrate an understanding that different shapes might share common attributes (e.g., four sides).
- can compare and classify two- dimensional shapes, particularly quadrilaterals.
- are able to partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole.
- can use common U.S. Customary and metric units for linear measurements (e.g., inches, feet, centimeters, and meters).
- can solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- understand the concept of area and can relate it to addition and multiplication to solve real-world problems.
- understand and can solve real-world and mathematical problems involving perimeter of polygons.

Geometric Measurement: *Measure and estimate lengths in standard units.*	**ABE2.GM.1**	Measure the length of an object by selecting and using appropriate tools (e.g., ruler, meter stick, yardstick, measuring tape). (2.MD.1)
	ABE2.GM.2	Measure the length of an object twice, using different standard length units for the two measurements; describe how the two measurements relate to the size of the unit chosen. Understand that depending on the size of the unit, the number of units for the same length varies. (2.MD.2)
	ABE2.GM.3	Estimate lengths (using US customary or metric units), compare the estimation to actual measurement, and justify the reasonableness of the answer. (2.MD.3)
	ABE2.GM.4	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (2.MD.4)
Geometric Measurement: *Relate addition and subtraction to length.*	**ABE2.GM.5**	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units (e.g., by using drawings, such as drawings of rulers, and equations with a symbol for the unknown number to represent the problem). (2.MD.5)
	ABE2.GM.6	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram. (2.MD.6)
Geometric Measurement: *Solve problems involving measurement.*	**ABE2.GM.7**	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes (e.g., representing the problem on a number line diagram). (3.MD.1)
	ABE2.GM.8	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units (e.g., by using drawings, such as a beaker with a measurement scale, to represent the problem). (3.MD.2)

Geometric Measurement: *Understand concepts of area and perimeter.*	**ABE2.GM.9**	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. (3.MD.5)
	ABE2.GM.10	Measure areas by counting unit squares (square cm, square m, square in, square ft., and using improvised units). (3.MD.6)
	ABE2.GM.11	Relate area to the operations of multiplication and addition. (3.MD.7)
	ABE2.GM.12	Find the area of a rectangle with whole number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. (3.MD.7a)
	ABE2.GM.13	Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. (3.MD.7b)
	ABE2.GM.14	Use tiling to show that the area of a rectangle with whole-number side lengths a and b + c is the sum of a • b and a • c. Use area models to represent the distributive property in mathematical reasoning. (3.MD.7c)
	ABE2.GM.15	Recognize area as additive. Find areas of linear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems. (3.MD.7d)
	ABE2.GM.16	Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas, or with the same area and different perimeters. (3.MD.8)
Geometric Figures: *Reason with shapes and their attributes.*	**ABE2.GM.17**	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (2.G.1)
	ABE2.GM.18	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *fourths*, *half of*, *third of, fourth of*, and describe the whole as *two halves, three thirds, or four fourths*. Recognize that equal shares of identical wholes need not have the same shape. (2.G.3)
	ABE2.GM.19	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples quadrilaterals that do not belong to any of these subcategories. (3.G.1)
	ABE2.GM.20	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 5 parts with equal area and describe the area of each part as 1/5 of the area of the shape. (3.G.2)*

Data Analysis and Statistics (DS)

Data and Probability: Students prepared to exit ABE 2

- are able to draw and interpret simple graphs (e.g., bar graphs, picture graphs, and number line diagrams) including scaled bar and picture graphs.
- can solve one- and two-step problems using scaled bar graphs.
- can generate measurement data by measuring lengths to the nearest half- and quarter-inch and display that data by making a line plot marked off in appropriate units.

Data and Statistics: *Represent and interpret data.*	**ABE2.DS.1**	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put- together, take-apart problems, and compare problems using information presented in a bar graph. (2.MD.10)
	ABE2.DS.2	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets*. (3.MD.3)
	ABE2.DS.3	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters. (3.MD.4)

ABE 3: Low Intermediate

The Mathematical Practices: Students prepared to exit ABE 3

- are able to decipher multi-step problems presented in a context
- can reason about and apply the correct units and the proper degree of precision to the results.
- can visualize a situation using diagrams or sketches.
- can see multiple strategies for solving a problem.
- can explain their processes and results.
- can recognize errors in the work and reasoning of others.
- can express themselves using mathematical terms and notation appropriate for the level
- can strategically select and use tools to aid in their work, such as pencil/paper, measuring devices, and/or technology.
- are able to see patterns and structure in sets of numbers and geometric shapes and use those insights to work more efficiently.

Number Sense and Operations (NO)

***Number Sense*:** Students prepared to exit ABE 3

- understand place value for both multi-digit whole numbers and decimals to thousandths
- use their understanding to read, write, compare, and round decimals.
- are able to use their place value understanding and properties of operations to fluently perform operations with multi-digit whole numbers and decimals.
- can find common factors, common multiples, and understand fraction concepts, including fraction equivalence and comparison.
- can add, subtract, multiply and divide with fractions and mixed numbers.
- are able to solve multi-step word problems posed with whole numbers and fractions, using the four operations.
- understand ratio concepts and can use ratio language to describe a relationship between two quantities, including the concept of a unit rate associated with a ratio.

Number Base Ten: *Generalize place value understanding for multi-digit whole numbers.* [NOTE: Level 3 expectations in this domain are for whole numbers less than or equal to 1,000,000.]	**ABE3.NO.1**	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (4.NBT.2)
	ABE3.NO.2	Use place value understanding to round multi-digit whole numbers to any place. (4.NBT.3)
	ABE3.NO.3	Fluently add and subtract multi-digit whole numbers using a standard algorithm. (4.NBT.4)
	ABE3.NO.4	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (4.NBT.5)
	ABE3.NO.5	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (4.NBT.6)

Number Base Ten: *Understand the place value system.*	**ABE3.NO.6**	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. (5.NBT.1)
	ABE3.NO.7	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. (5.NBT.2)
	ABE3.NO.8	Read, write, and compare decimals to thousandths. (5.NBT.3)
	ABE3.NO.9	Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (5.NBT.3a)
	ABE3.NO.10	Use place value understanding to round decimals to any place. (5.NBT.4)
Number Base Ten: *Perform operations with multi-digit whole numbers and with decimals to hundredths.*	**ABE3.NO.11**	Fluently multiply multi-digit whole numbers using the standard algorithm. *[NOTE: A "standard algorithm" might be any accepted algorithm that fits the experience and needs of the students.]* (5.NBT.5)
	ABE3.NO.12	Apply and extend understanding of division to find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. (5.NBT.6)
	ABE3.NO.13	Add, subtract, multiply, and divide decimals to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations. (5.NBT.7)

Fractions: *Extend understanding of fraction equivalence and ordering.* [NOTE: Level 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, *10, 12, and 100.]*	**ABE3.NO.14**	Explain why a fraction a/b is equivalent to a fraction $(n \cdot a)/(n \cdot b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to understand and generate equivalent fractions. (4.NF.1)
	ABE3.NO.15	Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model). (4.NF.2)
Fractions: *Apply and extend previous understanding of multiplication to multiply a whole number by a fraction.*	**ABE3.NO.16**	Understand a fraction a/b with $a > 1$ as a sum of unit fractions ($1/b$). (4.NF.3)
	ABE3.NO.17	Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (4.NF.3a)
	ABE3.NO.18	Decompose a fraction into a sum of fractions with the same denominator in more than one way (e.g., 3/8 = 1/8 + 1/8+1/8; 3/8 = 2/8 + 1/8; 2 1/8 = 1 + 1 + 1/8 + or 2 1/8 = 8/8 + 8/8 + 1/8). (4.NF.3b)
	ABE3.NO.19	Add and subtract mixed numbers with like denominators (e.g., by using properties of operations and the relationship between addition and subtraction and/or by replacing each mixed number with an equivalent fraction). (4.NF.3c)
	ABE3.NO.20	Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators (e.g., by using visual fraction models and equations to represent the problem). (4.NF.3d)
	ABE3.NO.21	Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 • (2/5) as 6 • (1/5), recognizing this product as 6/5. (In general,* $n \cdot (a/b) = (n \cdot a)/b$.) (4.NF.4b)
	ABE3.NO.22	Solve word problems involving multiplication of a fraction by a whole number (e.g., by using visual fraction models and equations to represent the problem). *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* (4.NF.4c)

Fractions: *Understand decimal notation for fractions, and compare decimal fractions.*	**ABE3.NO.23**	Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*. (4.NF.6)
	ABE3.NO.24	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual model). (4.NF.7)
Fractions: *Use equivalent fractions to add and subtract fractions.*	**ABE3.NO.25**	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. F*or example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general,* a/b + c/d = (ad + bc)/bd*.)* (5.NF.1)
	ABE3.NO.26	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2*. (5.NF.2)
Fractions: *Use previous understandings of multiplication and division to multiply and divide fractions.*	**ABE3.NO.27**	Interpret a fraction as division of the numerator by the denominator (*a/b = a ÷ b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers (e.g., by using visual fraction models or equations to represent the problem). *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* (5.NF.3)
	ABE3.NO.28	Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. (5.NF.4c)
	ABE3.NO.29	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number; explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a/b = (b • a) / (n • b)* to the effect of multiplying *a/b* by 1. (5.NF.5)
	ABE3.NO.30	Solve real-world problems involving multiplication of fractions and mixed numbers (e.g., by using visual fraction models or equations to represent the problem). (5.NF.6)
	ABE3.NO.31	Interpret division of a unit fraction by a non-zero whole number and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) • 4 = 1/3*. (5.NF.7a)

	ABE3.NO.32	Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 • (1/5) = 4.* (5.NF.7b)
	ABE3.NO.33	Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions (e.g., by using visual fraction models and equations to represent the problem). *For example, how much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?* (5.NF.7c)
Fractions: *Apply and extend previous understanding of multiplication and division to divide fractions by fractions.*	**ABE3.NO.34**	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions (e.g., by using visual fraction models and equations to represent the problem). *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) =* ad/bc*) How much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?* (6.NS.1)

Algebraic Thinking (AT)

Algebra: Students prepared to exit ABE 3

- are able to apply and extend their understanding of arithmetic to algebraic expressions, using a symbol to represent an unknown value.
- can write, evaluate, and interpret expressions and equations, including expressions that arise from formulas used in real-world problems.
- can solve real-world and mathematical problems by writing and solving simple one-variable equations.
- can write a simple inequality that represents a constraint or condition in a real-world or mathematical problem.
- can represent and analyze quantitative relationships between dependent and independent variables.

Operations and Algebraic Thinking: *Use the four operations with whole numbers to solve problems.*	**ABE3.AT.1**	Translate verbal statements of multiplicative comparisons to multiplication equations and vice versa (e.g., interpret 35 = 5 • 7 as 35 is the total number in 5 groups, each containing 7 objects, and is also the number of objects in 7 groups, each containing 5 objects). (4.OA.1)
	ABE3.AT.2	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (4.OA.3)
Operations and Algebraic Thinking: *Gain familiarity with factors and multiples.*	**ABE3.AT.3**	Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. (4.OA.4)

Operations and Algebraic Thinking: *Generate and analyze patterns.*	**ABE3.AT.4**	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain why the numbers will continue to alternate in this way.* (4.OA.5)
Operations and Algebraic Thinking: *Write and interpret numerical expressions.*	**ABE3.AT.5**	Use parentheses and brackets in numerical expressions, and evaluate expressions with these symbols (Order of Operations). (5.OA.1)
	ABE3.AT.6	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product*. (5.OA.2)
Operations and Algebraic Thinking: *Compute fluently with multi-digit numbers and find common factors and multiples.*	**ABE3.AT.7**	Fluently divide multi-digit numbers using a standard algorithm. (6.NS.2)
	ABE3.AT.8	Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation. (6.NS.3)
	ABE2.AT.9	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).* (6.NS.4)
Ratio and Proportion: *Understand ratio concepts and use ratio reasoning to solve problems.*	**ABE3.AT.10**	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* (6.RP.1)
	ABE3.AT.11	Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar" or "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* (6.PR.2)
Expressions and Equations: *Apply and extend previous understanding of arithmetic to algebraic expressions.*	**ABE3.AT.12**	Write and evaluate numerical expressions involving whole-number exponents. (6.EE.1)
	ABE3.AT.13	Write, read, and evaluate expressions in which letters stand for numbers. (6.EE.2)
	ABE3.AT.14	Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* (6.EE.2a)
	ABE3.AT.15	Identify parts of an expression using mathematical terms (e.g., sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* (6.EE.2b)

	ABE3.AT.16	Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6\ s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$*.* (6.EE.2c)
	ABE3.AT.17	Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* (6.EE.3)
	ABE3.AT.18	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.* (6.EE.4)
Expressions and Equations: *Reason about and solve one-variable equations and inequalities.*	**ABE3.AT.19**	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (6.EE.5)
	ABE3.AT.20	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. (6.EE.6)
	ABE3.AT.21	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all non-negative rational numbers. (6.EE.7)
	ABE3.AT.22	Write an inequality of the form $x > c$, $x < c$, $x \geq c$, or $x \leq c$ to represent a constraint or condition to solve mathematical problems and problems in real-world context. Recognize that inequalities have infinitely many solutions; represent solutions of such inequalities on number lines. (6.EE.8)
Expressions and Equations: *Represent and analyze quantitative relationships between dependent and independent variables.*	**ABE3.AT.23**	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.* (6.EE.9)

Geometry and Measurement (GM)		
Geometry: Students prepared to exit ABE 3 • have a basic understanding of the coordinate plane and can plot points (i.e., ordered pairs) and place polygons in the coordinate plane to solve real-world and mathematical problems. • can classify two-dimensional shapes and use formulas to determine the area of two-dimensional shapes such as triangles and quadrilaterals. • can determine the surface area of three-dimensional shapes composed of rectangles and triangles, and find the volume of right rectangular prisms. • are able to convert like measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m) and use these conversions to solve multi-step, real-world problems. • are also able to solve measurement word problems (such as those that involve area, perimeter, distance, time intervals, liquid volumes, mass, and money) that involve simple fractions or decimals.		
Geometric Measurement: *Understand concepts of angle and measure angles.*	**ABE3.GM.1**	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through one-degree angles is said to have an angle measure of *n* degrees. (4.MD.5)
	ABE3.GM.2	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. (4.MD.6)
	ABE3.GM.3	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems (e.g., by using an equation with a symbol for the unknown angle measure). (4.MD.7)
Geometric Measurement: *Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.*	**ABE3.GM.4**	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (4.MD.2)
	ABE3.GM.5	Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* (4.MD.3)

Geometric Measurement: *Convert like measurement units within a given measurement system.*	**ABE3.GM.6**	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems. (5.MD.1)
Geometric Measurement: *Understand concepts of volume and relate volume to multiplication and to addition.*	**ABE3.GM.7**	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. (5.MD.3)
	ABE3.GM.8	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft., and improvised units. (5.MD.4)
	ABE3.GM.9	Relate volume to the operations of multiplication and addition and solve mathematical problems and problems in real-world contexts involving volume. (5.MD.5)
	ABE3.GM.10	Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication). (5.MD.5a)
	ABE3.GM.11	Understand and use the formula, $V = lwh$, for rectangular prisms, to find volumes of right rectangular prisms with whole-number edge lengths to solve mathematical problems and problems in real-world contexts. (5.MD.5b)
	ABE3.GM.12	Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems. (5.MD.5c)

Geometric Figures: *Draw and identify lines and angles, and classify shapes by properties of their lines and angles.*	**ABE3.GM.13**	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. (4.G.1)
Geometric Figures: *Graph points on the coordinate plane to solve mathematical problems as well as problems in real-world context.*	**ABE3.GM.14**	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). (5.G.1)
	ABE3.GM.15	Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. (5.G.2)
Geometric Figures: *Classify two-dimensional figures into categories based on their properties.*	**ABE3.GM.16**	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles, and squares are rectangles, so all squares have four right angles.* (5.G.3)
Geometric Figures: *Solve mathematical problems and problems in real-world context involving area, surface area, and volume.*	**ABE3.GM.17**	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. (6.G.1)
	ABE3.GM.18	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. (6.G.3)
	ABE3.GM.19	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. (6.G.4)

Data Analysis and Statistics (DS)

Data and Probability: Students prepared to exit ABE 3

- have a basic conceptual understanding of statistical variability, including such concepts as center, spread, and the overall shape of a distribution of data.
- can present data using displays such as dot plots, histograms, and box plots.

Data and Statistics: *Develop understanding of statistical variability.*	**ABE3.DS.1**	Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center, spread, and overall shape. (6.SP.2)
	ABE3.DS.2	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (6.SP.3)
Data and Statistics: *Summarize and describe distributions.*	**ABE3.DS.3**	Display and interpret numerical data by creating plots on a number line including histograms, dot plots, and box plots. (6.SP.4)

ABE 4: High Intermediate

The Mathematical Practices: Students prepared to exit ABE 4

- are able to think critically, determine an efficient strategy (from among multiple possible strategies) for solving a multi-step problem, and persevere in solving challenging problems.
- can express themselves using the mathematical terms and notation appropriate to the level.
- are able to defend their findings and critique the reasoning of others.
- are accurate in their calculations and use estimation strategies to assess the reasonableness of their results.
- can create algebraic and geometric models and use them to answer questions and solve problems.
- can strategically select and use tools to aid in their work, such as pencil/paper, measuring devices, calculators, and/or spreadsheets.
- are able to see patterns and structure in number sets, data, expressions and equations, and geometric figures.

NOTE: For some standards, part of the requirement is addressed in Level 4 and the rest in Level 5. In those cases, the portion of the standard that is identified for this level is in blue font and underlined. [See ABE4.N.14, ABE4.N.15, ABE4.N.19 and ABE4.D.4.]

Number Sense and Operations (NO)		
Number Sense: Students prepared to exit ABE 4 • understand the rational number system, including how rational numbers can be represented on a number line and pairs of rational numbers can be represented on a coordinate plane. • can apply the concept of absolute value to find horizontal and vertical distances. • are able to apply the properties of integer exponents and evaluate, estimate, and compare simple square roots and cube roots. • understand ratio, rate, and percent concepts, as well as proportional relationships.		
The Number System: *Apply and extend previous understanding of numbers to the system of rational numbers.* [NOTE: Limit negative rational numbers to integers and fractions with denominators of 2, 3, 4, 5, 10.]	**ABE4.NO.1**	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. (6.NS.5)
	ABE4.NO.2	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes to represent points on the line and in the plane with negative number coordinates. (Including quadrants II, III, IV of the coordinate plane) (6.NS.6)
	ABE4.NO.3	Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself (e.g., –(–3) = 3, and that 0 is its own opposite). (6.NS.6a)
	ABE4.NO.4	Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. (6.N.6b)
	ABE4.NO.5	Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. (6.NS.6c)

	ABE4.NO.6	Understand ordering and absolute value of rational numbers. (6.NS.7)
	ABE4.NO.7	Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.* (6.NS.7a)
	ABE4.NO.8	Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 degrees Celsius > –7 degrees Celsius to express the fact that –3 degrees Celsius is warmer than –7 degrees Celsius.* (6.NS.7b)
	ABE4.NO.9	Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.* (6.NS.7c)
	ABE4.NO.10	Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.* (6.NS.7d)
	ABE4.NO.11	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. (6.NS.8)
The Number System: *Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers (except division by zero).*	**ABE4.NO.12**	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. (7.NS.1)
	ABE4.NO.13	Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* (7.NS.1a)
	ABE4.NO.14	Understand *p* + *q* as the number located a distance |*q*| from p, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. *[NOTE: In Level 4 focus on an understanding of using the number line for finding, or verifying, sums with positive and negative rational numbers, including those whose sum is zero. This standard will be fully addressed in Level 5.]* (7.NS.1b)
	ABE4.NO.15	Understand subtraction of rational numbers as adding the additive inverse, *p – q = p + (–q)*. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. *[NOTE: For Level 4 focus on defining "additive inverse" (opposite) and rewriting subtraction problems, particularly those involving negative numbers, as addition problems. This standard will be fully addressed in Level 5.]* (7.NS.1c)

	ABE4.NO.16	Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. (7.NS.2a)
	ABE4.NO.17	Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts (7.NS.2b)
	ABE4.NO.18	Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. (7.NS.2d)
Numerical Expressions: *Work with radicals and integer exponents.*	**ABE4.NO.19**	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \cdot 3^{(-5)} = 3^{(-3)} = (1/3)^3 = 1/27$. *[NOTE: For Level 4 focus on understanding the relationship between multiplication and division, as it applies to the properties of integer exponents: that a number with a negative exponent is equal to the reciprocal of the number to a positive exponent; and that multiplication by a number to a negative exponent is the same as division by the number to a positive exponent. Use the properties of integer exponents to rewrite numerical expressions involving exponents with a common base. This standard will be fully addressed in Level 5.]* (8.EE.1)
	ABE4.NO.20	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of perfect squares to 225 and cube roots of perfect cubes to 1000. Know that √2 is irrational. *[NOTE: For Level 4 focus on work with square and cube roots: use correct notation and evaluate roots of perfect squares and cubes. This standard will be fully addressed in Level 5.]* (8.EE.2)

Algebraic Thinking (AT)

Algebra: Students prepared to exit ABE 4

- understand the connections between proportional relationships, lines, and linear equations.
- understand numerical and algebraic expressions, and equations and are able to use them to solve real-world and mathematical problems.
- are able to analyze and solve linear equations and pairs of simultaneous linear equations.
- are able to define, interpret, and compare linear functions.

Expressions and Equations: *Use properties of operations to generate equivalent expressions.*	**ABE4.AT.1**	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. (7.EE.1)
	ABE4.AT.2	Rewrite an expression in different forms, and understand the relationship between the different forms and their meanings in a problem context. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* (7.EE.2)

Expressions and Equations: *Solve mathematical problems and problems in real-world context using numerical and algebraic expressions and equations.*	**ABE4.AT.3**	Use variables to represent quantities in mathematical problems and problems in real-world context, and construct simple equations and inequalities to solve problems. (7.EE.4)
Expressions and Equations: *Analyze and solve linear equations, inequalities, and pairs of simultaneous linear equations.*	**ABE4.AT.4**	Fluently solve linear equations and inequalities in one variable. (8.EE.7)
	ABE4.AT.5	Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. (8.EE.8a)
	ABE4.AT.6	Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* (8.EE.8b)
Ratio and Proportion: *Understand ratio concepts and use ratio reasoning to solve problems.*	**ABE4.AT.7**	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (6.RP.3)
	ABE4.AT.8	Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. (6.RP.3a)
	ABE4.AT.9	Solve unit rate problems, including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* (6.RP.3b)
	ABE4.AT.10	Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. (6.RP.3c)
	ABE4.AT.11	Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. (6.RP.3d)

Ratio and Proportion: *Analyze proportional relationships and use them to solve real-world and mathematical problems.*	**ABE4.AT.12**	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.* (7.RP.1)
	ABE4.AT.13	Recognize and represent proportional relationships between quantities. *[For Level 4 focus on recognizing when relationships between quantities, represented either algebraically or graphically, are proportional.]* (7.RP.2)
Functions: *Define, evaluate, and compare functions.*	**ABE4.AT.14**	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (8.F.1)
	ABE4.AT.15	Compare properties of two linear functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (8.F.2)
	ABE4.AT.16	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.* (8.F.3)

Geometry and Measurement (GM)

Geometry**:** Students prepared to exit ABE 4

- can solve real-world and mathematical problems that involve angle measure, circumference, and area of 2-dimensional figures.
- are able to solve problems involving scale drawings of 2-dimensional geometric figures.
- understand the concepts of congruence and similarity with respect to 2-dimensional figures.
- understand the Pythagorean theorem and can apply it to determine missing lengths in right triangles.

Geometric Measurement: *Draw, construct, and describe geometrical figures, and describe the relationships between them.*	**ABE4.GM.1**	Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. (7.G.1)

Geometric Measurement: *Solve mathematical problems and problems in real-world context involving circle measures.*	**ABE4.GM.2**	Understand and use the formulas for the area and circumference of a circle to solve problems; give an informal explanation of the relationship between the circumference and area of a circle. (7.G.4)
Geometric Figures: *Understand congruence and similarity.*	**ABE4.GM.3**	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (8.G.2)
	ABE4.GM.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two- dimensional figures, describe a sequence that exhibits the similarity between them. (8.G.4)
Geometric Figures: *Understand and apply the Pythagorean Theorem.*	**ABE3.GM.5**	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (8.G.7)

Data Analysis and Statistics (DS)

Data and Probability: Students prepared to exit ABE 4

- can summarize and describe numerical data sets in relation to their context, including determining measures of center and variability and describing patterns and/or striking deviations from patterns.
- understand and can apply the concept of chance, or probability.
- are able to use scatter plots for bivariate measurement data to describe patterns of association between two quantities (such as clustering, outliers, positive or negative association, linear or non-linear association).

Data and Statistics: *Summarize and describe distributions.*	**ABE4.DS.1**	Summarize numerical data sets in relation to their context by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (6.SP.5)
Data and Statistics: *Use random sampling to draw inferences about a population.*	**ABE4.DS.2**	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. (7.SP.1)
	ABE4.DS.3	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* (7.SP.2)
Data and Statistics: *Investigate patterns of association in bivariate data.*	**ABE4.DS.4**	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. *[NOTE: For ABE 4 focus on constructing scatter plots from measurement data, recognizing when there appears to be a linear association, and recognizing when a linear association is negative or positive. This standard will be fully addressed in ABE 5.]* (8.SP.1)

Probability: *Investigate chance processes and develop, use, and evaluate probability models.*	**ABE4.DS.5**	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. (7.SP.5)
	ABE4.DS.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* (7.SP.6)
	ABE4.DS.7	Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. (7.SP.8a)
	ABE4.DS.8	Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. (7.SP.8b)

ABE 5: Low Adult Secondary

The Mathematical Practices*:* Students prepared to exit ABE 5

- are able to think critically,
- determine an efficient strategy (from among multiple possible strategies) for solving a multi-step problem, and
- persevere in solving challenging problems.
- can reason quantitatively, including using units as a way to solve problems.
- are able to defend their findings and critique the reasoning of others.
- are accurate in their calculations and use estimation strategies to assess the reasonableness of their results.
- can create algebraic and geometric models and use them to answer questions and solve problems.
- can strategically select and use tools to aid in their work, such as graphing calculators, spreadsheets, and/or computer software.
- are able to make generalizations based on patterns and structure they discover in number sets, data, expressions and equations, and geometric figures and use these insights to work more efficiently.

NOTE: Some standards in this level were partially addressed in Levels 4. In those cases, the portion of the standard that is identified for this level is in blue font and underlined. [See ABE5.N.1, ABE5.N.2, ABE5.N.6, and ABE5.D.3.]

Number Sense and Operations (NO)

Number Sense: Students prepared to exit ABE 5

- can reason about and solve real-world and mathematical problems that involve the four operations with rational numbers.
- can apply the concept of absolute value to demonstrate on a number line their understanding of addition and subtraction with negative and positive rational numbers.
- can apply ratio and percent concepts, including using rates and proportional relationships to solve multi-step real-world and mathematical problems.

The Number System: *Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers (except division by zero).*	**ABE5.NO.1**	Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. *[NOTE: This standard was partially addressed in Level 4. For Level 5 focus on interpreting rational number sums by identifying the numbers as quantities in real-world situations.]* (7.NS.1b)
	ABE5.NO.2	Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. *[NOTE: This standard was partially addressed in Level 4. For Level 5 focus on using absolute value to develop an efficient algorithm for subtraction problems that involve negative rational numbers and applying the algorithm to real-world problems.]* (7.NS.1c)

	ABE5.NO.3	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. (7.NS.2)
	ABE5.NO.4	Apply properties of operations as strategies to multiply and divide rational numbers. (7.NS.2c)
	ABE5.NO.5	Solve mathematical problems and problems in real-world context involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions where $a/b \div c/d$ when *a, b, c*, and *d* are all integers and *b*, *c*, and $d \neq 0$. (7.NS.3)
Numerical Expressions: *Work with integer exponents.*	**ABE5.NO.6**	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $8 \cdot 2^{(-5)} = 2^3 \cdot 2^{(-5)} = (2)^{-2} = 1/2^2 = 1/4$. [NOTE: This standard was partially addressed in Level 4. For Level 5 focus on using the properties of exponents to rewrite numerical expressions involving integer exponents (negative and positive) in equivalent forms and to solve problems that involve integer exponents (positive and negative). Numerical expressions with different bases must be able to be converted to the same base.]* (8.EE.1)
	ABE5.NO.7	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as $3 \cdot 10^8$ and the population of the world as $7 \cdot 10^9$, and determine that the world population is more than 20 times larger.* (8.EE.3)
	ABE5.NO.8	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. (8.EE.4)
The Number System: *Understand that there are irrational numbers, and approximate them using rational numbers.*	**ABE5.NO.9**	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion. Know that numbers whose decimal expansions do not terminate in zeros or in a repeating sequence of fixed digits are called irrational. (8.NS.1)
	ABE5.NO.10	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.* (8.NS.2)

Algebraic Thinking (AT)

Algebra: Students prepared to exit ABE 5

- are able to use algebraic and graphical representations to solve real-world and mathematical problems, involving linear equations, inequalities, and pairs of simultaneous linear equations.
- are able to use linear functions to describe, analyze, and model linear relationships between quantities.

Expressions and Equations: *Work with integer exponents.*	**ABE5.AT.1**	Multiply and divide expressions involving exponents with a common base. *[NOTE: Expressions may be algebraic; however, the exponents must be integers.]* (No CCR tag)
Expressions and Equations: *Solve mathematical problems and problems in real-world context using numerical and algebraic expressions and equations.*	**ABE5.AT.2**	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* (7.EE.3)
	ABE5.AT.3	Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* (7.EE.4a)
	ABE5.AT.4	Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.* (7.EE.4b)

Expressions and Equations: *Analyze and solve linear equations, inequalities, and pairs of simultaneous linear equations.*	**ABE5.AT.5**	Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). (8.EE.7a)
	ABE5.AT.6	Solve linear equations and inequalities with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. (8.EE.7b)
	ABE5.AT.7	Analyze and solve a system of two linear equations. (8.EE.8)
	ABE5.AT.8	Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, consider these two options for getting paid for a job: 1) You earn $12/hour for all hours worked in each week or 2) You earn $9/hour for the first 40 hours plus 1.5 times the hourly rate for any hours worked over 40. Determine the number of hours at which the two pay options are equal.* (8.EE.8c)
Expressions and Equations: *Understand the connections between proportional relationships, lines, and linear equations.*	**ABE5.AT.9**	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (8.EE.5)
	ABE5.AT.10	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $(0, b)$. (8.EE.6)
Ratio and Proportion: *Analyze proportional relationships and use them to solve real-world and mathematical problems.*	**ABE5.AT.11**	Recognize and represent proportional relationships between quantities. *[For Level 5 focus on representing proportional relationships between quantities when given verbally, algebraically, or graphically; and translating between representations.]* (7.RP.2)
	ABE5.AT.12	Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). (7.RP.2a)
	ABE5.AT.13	Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. (7.RP.2b)
	ABE5.AT.14	Represent proportional relationships by equations. F*or example, if total cost* **t** *is proportional to the number* **n** *of items purchased at a constant price* **p**, *the relationship between the total cost and the number of items can be expressed as* **t = pn**. (7.RP.2c)
	ABE5.AT.15	Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate. (7.RP.2d)
	ABE5.AT.16	Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* (7.RP.3)

Functions: *Use functions to model relationships between quantities.*	**ABE5.AT.17**	Given a description of a situation, generate a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or a graph. Track how the values of the two quantities change together. Interpret the rate of change and initial value of a linear function in terms of the situation it models, its graph, or its table of values. (8.F.4)
	ABE5.AT.18	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (8.F.5)

Geometry and Measurement (GM)

Geometry: Students prepared to exit ABE 5

- can solve real-world and mathematical problems that involve volume and surface area of 3-dimensional geometric figures.
- can use informal arguments to establish facts about various angle relationships such as the relationships between angles created when parallel lines are cut by a transversal.
- apply the Pythagorean theorem to determine lengths in real-world contexts and distances in the coordinate plane.

Geometric Measurement: *Solve mathematical problems and problems in real-world context involving angle measure, area, surface area, and volume.*	**ABE5.GM.1**	Use facts about supplementary, complementary, vertical, and adjacent angles in multi-step problems to write and solve simple equations for an unknown angle in a figure. (7.G.5)
	ABE5.GM.2	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (7.G.6)
Geometric Figures: *Understand and apply the Pythagorean Theorem.*	**ABE5.GM.3**	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (8.G.8)

Data Analysis and Statistics (DS)		
***Data and Probability*: Students prepared to exit ABE 5 • can use random sampling to draw inferences about a population. • are able to draw informal comparative inferences about two populations using measures of center and measures of variability for numerical data from random samples. • can develop, use, and evaluate probability models. • are able to use scatter plots for bivariate measurement data to interpret patterns of association between two quantities (such as clustering, outliers, positive or negative association, linear or non-linear association) and a 2-way table to summarize and interpret bivariate categorical data.**		
Data and Statistics: *Draw informal comparative inferences about two populations.*	**ABE5.DS.1**	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* (7.SP.3)
	ABE5.DS.2	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* (7.SP.4)
Data and Statistics: *Investigate patterns of association in bivariate data.*	**ABE5.DS.3**	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. *[NOTE: This standard was partially addressed in Level 4. For Level 5 focus on interpreting scatter plots: describing the patterns that affect the association and the features of the data that appear to be responsible for those patterns; distinguishing between linear and nonlinear associations; and investigating what clusters and outliers reveal about the data.]* (8.SP.1)
	ABE5.DS.4	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (8.SP.2)
	ABE5.DS.5	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a botany experiment, interpret a slope of 1.5 cm/hr. as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* (8.SP.3)

	ABE5.DS.6	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data on how many days in the last year were of below-average temperatures and how many days were overcast. Is there evidence that overcast days also had below-average temperatures?* (8.SP.4)
Probability: *Investigate chance processes and develop, use, and evaluate probability models.*	**ABE5.DS.7**	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. (7.SP.7)
	ABE5.DS.8	Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a six-sided die is rolled, find the probability that a number greater than three will come up and that an even number will come up.* (7.SP.7a)
	ABE5.DS.9	Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* (7.SP.7b)
	ABE5.DS.10	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. (7.SP.8)
	ABE5.DS.11	Understand that the probability of two disjoint events occurring is the sum of the two individual probabilities. (No CCR tag)

ABE 6: High Adult Secondary

The Mathematical Practices: Students prepared to exit ABE 6

- are able to think critically,
- make assumptions based on a situation,
- select an efficient strategy from multiple possible problem- solving strategies,
- plan a solution pathway, and
- make adjustments as needed when solving problems.
- persevere in solving challenging problems, including considering analogous, simpler problems as a way to solving a more complex one.
- can reason quantitatively, including through the use of units, and
- can express themselves using the precise definitions and mathematical terms and notation appropriate to the level.
- are accurate in their calculations,
- use an appropriate level of precision in finding solutions and reporting results, and
- use estimation strategies to assess the reasonableness of their results.
- are able to make conjectures,
- use logic to defend their conclusions, and
- can detect faulty thinking and errors caused by improper use of technology.
- can create algebraic and geometric models and use them to
    - answer questions,
    - interpret data,
    - make predictions, and
    - solve problems.
- can create algebraic and geometric models and use them to
    - answer questions,
    - interpret data,
    - make predictions, and
    - solve problems.
- can strategically select and use tools, such as
    - measuring devices,
    - calculators,
    - spreadsheets, and/or
    - computer software, to aid in their work.
- are able to see patterns and structure in calculations, expressions, and equations and make connections to algebraic generalizations, which they use to work more efficiently.

Modeling:

In order to help students experience the connection between classroom mathematics and real-world situations, teachers need to assist them in understanding and applying the modeling process. This is particularly important at ABE 6 since students now have enough mathematics knowledge to begin using the full modeling cycle. The following explanation of modeling is taken from Arizona's College and Career Ready Standards – Mathematics – High School (p. 61).

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing the total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other real-world situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the devised models depend on multiple factors:

- How precise an answer do we want or need?
- What aspects of the situation do we most need to understand, control, or optimize?
- What resources of time and tools do we have?

The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, when a model of bacterial growth makes more vivid the explosive growth of the exponential function.[7]

Since modeling with mathematics is a practice that is best interpreted in connection with mathematical content (see MP4), there are no isolated standards included that specifically require modeling.

To ensure the connection between content and modeling is made clear, an M is used as a modeling indicator at the end of the Level 6 standards that are most appropriate for application of the modeling cycle. Each M is also a hyperlink to the description of Modeling provided above.

Number Sense and Operations (NO)

Number Sense: Students prepared to exit ABE 6

- have extended their number sense to include irrational numbers, radicals, and rational exponents
- understand and use the set of real numbers.
- are able to assess the reasonableness of calculation results based on the limitations of technology or given units and quantities and give results with the appropriate degree of precision.

The Real Number System: *Extend the properties of exponents to rational exponents.*	**ABE6.NO.1**	Rewrite expressions involving radicals and rational exponents using the properties of exponents. (N-RN.2)
Quantities: *Reason quantitatively and use units to solve problems.*	**ABE6.NO.2**	Use units to understand problems and to guide the solution of multi-step problems, including some in real-world contexts; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (N-Q.1)
	ABE6.NO.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities utilizing real-world context. (N-Q.3)

[7] Arizona Department of Education – High Academic Standards for Students, Arizona's College and Career Ready Standards – Mathematics

Algebraic Thinking (AT)

Algebra: Students prepared to exit ABE 6

- understand the structure of expressions and can use that structure to rewrite linear, exponential, and quadratic expressions.
- can add, subtract, and multiply polynomials that involve linear and/or quadratic expressions.
- are also able to create linear equations and inequalities and quadratic and simple exponential equations to represent relationships between quantities and can represent constraints by linear equations or inequalities, or by systems of linear equations and/or inequalities.
- can interpret the structure of polynomial and rational expressions and use that structure to identify ways to rewrite and operate accurately with them.
- can add, subtract, and multiply polynomials that extend beyond quadratics.
- are able to rearrange formulas to highlight a quantity of interest, for example rearranging Ohm's law, V = IR, to highlight resistance R.
- are also able to create equations and inequalities representing relationships between quantities, including those that extend beyond equations or inequalities arising from linear, quadratic, and simple exponential functions to include those arising from simple rational functions.
- are able to use these equations/inequalities to solve problems both algebraically and graphically.
- can solve linear equations and inequalities; systems of linear equations; quadratic, simple rational, and radical equations in one variable; and recognize how and when extraneous solutions may arise.

Students prepared to exit ABE 6 also

- have a basic understanding of functions.
- can use function notation properly.
- use such notation to write a function describing a relationship between two quantities.
- are able to evaluate functions for inputs in their domains and interpret linear, quadratic, and exponential functions that arise in applications in terms of the context.
- are able to construct, graph, compare, and interpret functions (including, but not limited to, linear, quadratic, and exponential).
- can sketch graphs given a verbal description of the relationship and identify and interpret key features of the graphs of functions that arise in applications in a context.
- are able to select or define a function that appropriately models a relationship and to compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal description).

Seeing Structure in Expressions: *Interpret the structure of expressions.*	**ABE6.AT.1**	Interpret expressions that represent a quantity in terms of its context. **M** (A-SSE.1)
	ABE6.AT.2	Interpret parts of an expression, such as terms, factors, and coefficients. **M** (A-SSE.1a)
	ABE6.AT.3	Use structure to identify ways to rewrite numerical and polynomial expressions. Focus on polynomial multiplication and factoring patterns. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$. (A-SSE.2)

Seeing Structure in Expressions: *Write expressions in equivalent forms to solve problems.*	**ABE6.AT.4**	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Include problem-solving opportunities utilizing real-world context and focus on expressions with rational exponents. (A-SSE.3)
Arithmetic with Polynomial Expressions: *Perform arithmetic operations on polynomials.*	**ABE6.AT.5**	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. [*NOTE: Emphasis should be on operations with polynomials.]* (A-APR.1)
Arithmetic with Polynomials Expressions: *Understand the relationship between zeros and factors of polynomials.*	**ABE6.AT.6**	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Focus on quadratic and cubic polynomials in which linear and quadratic factors are available. (A-APR.3)
Creating Equations: *Create equations that describe numbers or relationships.*	**ABE6.AT.7**	Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on equations and inequalities that are linear, quadratic, or exponential. **M** (A-CED.1)
	ABE6.AT.8	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **M** (A-CED.2)
	ABE6.AT.9	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* **M** (A-CED.3)
	ABE6.AT.10	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.)* **M** (A-CED.4)

Reasoning with Equations and Inequalities: *Understand solving equations as a process of reasoning and explain the reasoning.*	**ABE6.AT.11**	Explain each step in solving linear and quadratic equations as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (A-REI.1)
Reasoning with Equations and Inequalities: *Solve equations and inequalities in one variable.*	**ABE6.AT.12**	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (A-REI.3)
	ABE6.AT.13	Solve quadratic equations in one variable. (A-REI.4)
	ABE6.AT.14	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - k)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. (A.REI.4a)
	ABE6.AT.15	Solve quadratic equations by inspection (e.g., $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Focus on solutions for quadratic equations that have real roots. Include cases that recognize when a quadratic equation has no real solutions. (A-REI.4b)
Reasoning with Equations and Inequalities: *Solve systems of equations.*	**ABE6.AT.16**	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (A-REI.6)
Reasoning with Equations and Inequalities: *Represent and solve equations and inequalities graphically.*	**ABE6.AT.17**	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A-REI.10)
	ABE6.AT.18	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and y=g(x) intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Focus on cases where $f(x)$ and/or $g(x)$ are linear, absolute value, quadratic, and exponential functions. (A-REI.11)
Interpreting Functions: *Understand the concept of a function and use function notation.*	**ABE6.AT.19**	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. (F-IF.1)
	ABE6.AT.20	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (F-IF.2)

Interpreting Functions: *Interpret functions that arise in applications in terms of the context.*	**ABE6.AT.21**	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums. Focus on linear, absolute value, quadratic, exponential and piecewise-defined functions (limited to the aforementioned functions). (F-IF.4)
	ABE6.AT.22	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* h(n) *gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. **M** (F-IF.5)
	ABE6.AT.23	Calculate and interpret the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context. Focus on linear, absolute value, quadratic, and exponential functions. (F-IF.6)
Interpreting Functions: *Analyze functions using different representations.*	**ABE6.AT.24**	Graph functions expressed symbolically and show key features of the graph by hand in simple cases, and using technology for more complicated cases. **M** (F-IF.7)
	ABE6.AT.25	Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* (F-IF.8b)
	ABE6.AT.26	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (F-IF.9)
Building Functions: *Build a function that models a relationship between two quantities.*	**ABE6.AT.27**	Write a function that describes a relationship between two quantities. **M** (F-BF.1)
Linear, Quadratic, and Exponential Models: *Construct and compare linear, quadratic, and exponential models and solve problems.*	**ABE6.AT.28**	Distinguish between situations that can be modeled with linear functions and with exponential functions. **M** (F-LE.1)
	ABE6.AT.29	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. **M** (F-LE.1a)
	ABE6.AT.30	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **M** (F-LE.1b)
	ABE6.AT.31	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **M** (F-LE.1c)
	ABE6.AT.32	Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. (F-LE.3)

Linear, Quadratic, and Exponential Models: *Interpret expressions for functions in terms of the situation they model.*	**ABE6.AT.33**	Interpret the parameters in a linear or exponential function with integer exponents utilizing real-world context. (F-LE.5)

Geometry and Measurement (GM)

Geometry: Students prepared to exit ABE 6

- can solve problems involving similarity and congruence criteria for triangles.
- can use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- can apply the concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTU's per cubic foot).

Congruence: *Experiment with transformations in the plane.*	**ABE6.GM.1**	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (G-CO.1)
Similarity, Right Triangles, and Trigonometry: *Prove theorems involving similarity.*	**ABE6.GM.2**	Use congruence and similarity criteria to prove relationships in geometric figures and solve problems utilizing real-world context. (G-SRT.5)
Expressing Geometric Properties with Equations: *Use coordinates to prove geometric theorems algebraically.*	**ABE6.GM.3**	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles. (G-GPE.7)
Geometric Measurement and Dimension: *Explain volume formulas and use them to solve problems.*	**ABE6.GM.4**	Analyze and verify the formulas for the volume of a cylinder, pyramid, and cone. (G-GMD.1)
	ABE6.GM.5	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems utilizing real-world context. **M** (G-GMD.3)

Modeling with Geometry: *Apply geometric concepts in modeling situations.*	**ABE6.GM.6**	Use geometric shapes, their measures, and their properties to describe objects utilizing real-world context. (G-MG.1)
	ABE6.GM.7	Apply geometric methods to solve design problems utilizing real-world context. (G-MG.3)

Data Analysis and Statistics (DS)

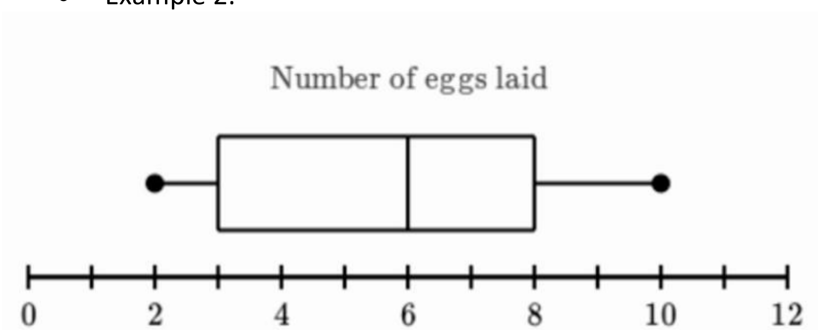
***Data and Probability*:** Students prepared to exit ABE 6

- can summarize, represent, and interpret data based on two categorical and quantitative variables, including by using frequency tables.
- can compare data sets by looking at commonalities and differences in shape, center, and spread.
- can recognize possible associations and trends in data, particularly in linear models, and distinguish between correlation and causation.
- interpret one- and two-variable data, including those with linear and non-linear relationships.
- interpret the slope (rate of change) and intercept (constant term) for a line of best fit and in the context of the data. They understand and account for extreme points of data in their analysis and interpret relative frequencies (joint, marginal and conditional).

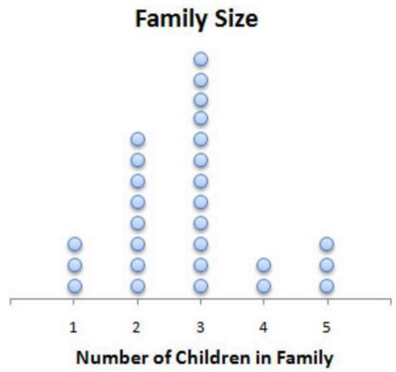
Interpreting Categorical and Quantitative Data: *Summarize, represent, and interpret data on a single count or measurement variable.*	**ABE6.DS.1**	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (S-ID.3)
Interpreting Categorical and Quantitative Data: *Summarize, represent, and interpret data on two categorical and quantitative variables.*	**ABE6.DS.2**	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (S-ID.5)
	ABE6.DS.3	Represent data on two quantitative variables on a scatter plot, and describe how the quantities are related. (S-ID.6)
	ABE6.DS.4	Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Focus on linear models. (S-ID.6a)
	ABE6.DS.5	Fit a linear function for a scatter plot that suggests a linear association. (S-ID.6c)

Interpreting Categorical and Quantitative Data: *Interpret models.*	**ABE6.DS.6**	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (S-ID.7)
	ABE6.DS.7	Distinguish between correlation and causation. (S.ID.9)
Making Inferences and Justifying Conclusions: *Make inferences and justify conclusions from experiments, and observational studies.*	**ABE6.DS.8**	Recognize the purposes of and differences between designed experiments, sample surveys and observational studies. (S-IC.3)
	ABE6.DS.9	Use data from a sample survey to estimate a population mean or proportion; recognize that estimates are unlikely to be correct and the estimates will be more precise with larger sample sizes. (S-IC.4)
	ABE6.DS.10	Evaluate reports based on data. (S-IC.6)

Mathematics Glossary

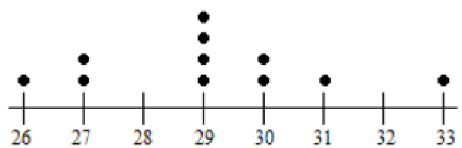
Additive inverses	Two numbers whose sum is 0. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.
Algorithm	a set of instructions/steps used to solve a problem or to reliably and consistently obtain a desired result
Array	an arrangement of objects or elements organized into rows and columns, or a set of objects or elements organized in a specific pattern
Associative property	• Addition: changing the grouping of terms in a sum without changing the sum. For any numbers *a*, *b*, and *c*: $(a + b) + c = a + (b + c)$ • Multiplication: changing the grouping of factors in a product without changing the product. For any numbers *a*, *b*, and *c*: $(a \cdot b)c = a(b \cdot c)$
Bivariate data	data that deals with two variables; pairs of linked numerical observations • Example: a list of the height and weight for each player on a football team.
Box plot	a method of visually displaying a univariate distribution of data values by using the median, quartiles, and extremes of the data set • Example 1: A box shows the middle 50% of the data. • Example 2: 

Clustering	collections of data elements in a display that are in close proximity relative to the context
Commutative property	• Addition: the addition of two terms in any order obtains the same sum. For any numbers *a* and *b*: $a + b = b + a$ • Multiplication: the multiplication of two terms in any order obtains the same product. For any numbers *a* and *b*: $a \cdot b = b \cdot a$
Complex fraction	a fraction that has a fractional numerator, denominator, or both. Examples: $\frac{a+\frac{1}{b}}{a-\frac{1}{b}}$ or $\frac{\frac{x^2-25}{y}}{x^3-5x^2}$ or $\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{4}+\frac{2}{3}}$
Composite figure	a geometric figure that is composed of two or more simpler figures
Congruent	geometric figures or angles that have the same size and shape; all corresponding sides and angles are the same size (congruent)
Decomposing	breaking numbers into their components or smaller parts
Dilation	a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor • The image is similar but not congruent to the pre-image.
Distributive property	Multiplication over addition: if *a*, *b*, and *c* are any signed numbers: $a(b + c) = ab + ac$
Domain	the set of independent values (x-values) for which a function or relation is defined

Dot plot	a method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line (Also see: Line plot.) Example: 
Equation	a mathematical statement composed of numbers, variables, operators (+, -, •, /) and an equal sign (=); any number sentence that states that two expressions have the same value
Expression	A mathematical phrase composed of variables, constants, and operations but without a symbol of equality or inequality (=, ≠, <, ≤, >, ≥).
Expanded form	a multi-digit number that is written as a sum of each digit multiplied by the power of 10 associated with the digit's place value. Example: *643 = (6 • 100) + (4 • 10) + (3 • 1)*
Exponent	a small number that is written above and to the right of a number or variable to show how many times the number or variable is to be multiplied by itself
First quartile	For a data set with median M, the first quartile is the median of the data values less than M; the number in a data set for which 25% of the data are less than that number. • Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6. See also: median, third quartile, interquartile range.

Fluency	the ability to perform calculations efficiently, accurately, flexibly, and appropriately • Students are able to choose flexibly among methods and strategies to solve contextual and mathematical problems; they understand and are able to explain their approaches; and they are able to produce accurate answers efficiently. • Efficiency—carries out calculations easily and without unnecessary steps or effort, keeps track of sub-problems, and makes use of intermediate results to solve the problem. • Accuracy—reliably produces a correct answer and checks the results in terms of the context. • Flexibility —knows more than one approach, chooses a viable strategy, and uses one method to solve and another method to check. • Appropriate Application—knows when to apply a particular procedure.[8]
Function	a relationship between two sets of numbers where for each value of the independent variable set (input) • There is only one value in the dependent variable set (output).
Inequality	a mathematical statement composed of variables, constants, operators (+, -, x, /), and a symbol of inequality (<, ≤, >, ≥) that compares two expressions, stating which is greater or less than the other
Integers	all positive and negative whole numbers, including zero
Interquartile range	a measure of variation in a set of numerical data, determined by finding the difference between the first and third quartiles of the data set; a way to describe the spread for a set of data • Example: For the data set {1, 3, <u>6</u>, 7, 10, 12, 14, <u>15</u>, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.
Irrational numbers	a set of real numbers that cannot be expressed as a ratio of two integers (i.e., π, √2)

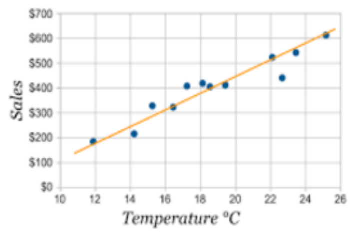
[8] National Council of Teachers of Mathematics. (2000). Principles and Standards for School Mathematics. Reston, VA: NCTM.

Line plot	a method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line (Also known as a dot plot.[4]) • Example: Line Plot: Number of Students per Classroom 
Mean (arithmetic)	a measure of center in a set of numerical data computed by adding the values in a list and then dividing by the number of values in the list; also called the average of the numbers in the data set. • Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21 (sum = 210; 210/10 = 21).
Median	a measure of center in a set of numerical data determined by finding the value appearing in the center of the list when the numbers are placed in increasing order—or the mean of the two central values, if the list contains an even number of values. • Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.
Order of Operations	rules that dictate the sequence the four calculations is done in an equation
Outlier	a data point that is distinctly separate from the rest of the data in a set
Percent rate of change	a rate of change over an interval of time expressed as a percent. Example: If a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year
Probability	a number between 0 and 1 used to quantify likelihood that a given event will have certain outcomes • Example: getting heads when tossing a coin, selecting a woman from a random group of people, hitting the bullseye of a target, or contracting a medical condition
Probability model	a mathematical representation of random phenomena, which assigns probabilities to outcomes of a chance event by examining the nature of the event. The set of all possible outcomes is called the sample space, and the sum of the probabilities of all events in the sample space is 1.

Properties	mathematical principles that are always true (See Tables 1-3)
Radical	the symbol for the indicated root of a number; an expression containing the root symbol. Also see: root. (Example: $\sqrt{\ }$, $\sqrt[3]{\ }$, $\sqrt[4]{\ }$)
Random sampling	A sampling method in which all members of a group (population) have an equal and independent chance of being selected[9]
Range	● <u>For data</u>: a measure of spread in a set of numerical data computed by finding the difference between the smallest and largest numbers in a data set ● <u>For a function or relation</u>: the set of dependent values (*y*-values) assumed by a function or relation over the set of all permitted values for the independent variable (*x*-values)
Ratio	a relationship in quantity, amount, or size between two or more values. ● Example: If the number of women in a group is 3 times that of the men, then the ratio of women to men is 3:1.
Rational number	a number that can be expressed as *a/b* or *-a/b* for some fraction *a/b*, where *a* and *b* are integers and $b \neq 0$) ● The rational numbers include the integers.
Reciprocals	two numbers whose product is 1. Example: 3/4 and 4/3 are reciprocals of one another because $3/4 \cdot 4/3 = 4/3 \cdot 3/4 = 1$ (Also called: Multiplicative Inverse)
Reflection	a transformation in which a geometric figure is reflected across a line, creating a mirror image[10] ● Congruence is preserved (i.e., the image and pre-image are congruent).
Relative frequency	The number of times that the event occurs during experimental trials, divided by the total number of trials conducted.
Root	● A number that produces the specified number when it is multiplied by itself the number of times designated by the given degree (uses a radical). [e.g., 16 = 4 or -4 but $\sqrt[4]{16} = 2^2$ or $(-2)^2$] ● A solution for an equation that places an expression equal to zero. Example: The roots of the equation $x^2 + 2x - 3 = 0$ are -3 and 1. (Also see definition of *zeros of a function*.)

[9] Web Finance Inc. (2017). Retrieved from http://www.businessdictionary.com/

[10] Mathwords: Terms and Formulas from Beginning Algebra to Calculus. (2014, July 28). Retrieved from http://www.mathwords.com/

Rotation	a transformation in which an object or coordinate system is turned about a fixed point; congruence is preserved (i.e., the image and pre-image are congruent)
Scatter plot	a graph in the coordinate plane representing a set of bivariate data by plotting the two variables (ordered pairs) in a coordinate plane • Example: The temperatures and sales for ice cream cones on different days could be displayed on a scatter plot.[11] • Example: 
Scale factor	a number that is used as a multiplier in order to "scale" a quantity (a scale factor that is between 0 and 1 is used to "scale down" and one that is greater than 1 to "scale up"); the ratio of any two corresponding lengths in two similar geometric figures
Spread	a measure of variation in a set of numerical data that describes how far apart are the values in the set (See also: range, for data, and interquartile range.)
Similar	geometric figures or angles that have the same shape but are of different size • All corresponding angles are congruent and corresponding sides are proportional.
Systems of equations	a set of two or more equations that must all be true for the same value(s) of their variables (also called simultaneous equations). **Note:** A system of equations can only be solved if the number of different equations is greater than or equal to and the number of variables.

[11] Adapted from Wisconsin Department of Public Instruction, op. Cit.

Systems of inequalities	a set of two or more inequalities that must all be true for the same value(s) (also called simultaneous inequalities). **Note:** The solution set for a system of inequalities will be the intersection of the sets of solutions for each inequality in the system.
Third quartile	For a data set with median M, the third quartile is the median of the data values greater than M; the number in a data set for which 75% of the data are less than that number. • Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. See also: median, first quartile, interquartile range.
Translation	a transformation in which every point on a figure moves a given distance in a given direction; congruence is preserved (i.e., the image and pre-image are congruent)
Univariate data	data that deals with a single variable; a collection of single values (Also see Bivariate data.)
Variable	A letter or symbol used to represent an unknown numerical value in a mathematical statement
Zeros of a function	the independent variable value(s) that make the function equal zero, or the values for *x* where *f(x)* = 0 Note: In the graph of a function, it is where the graph intercepts the *x*-axis. (Also see second definition of *root*.)

Appendix A: Educational Functioning Level Descriptions

The descriptions below have been taken from Implementation Guidelines: Measures and Methods for the National Reporting System for Adult Education, Appendix E (OCTAE, Feb. 2016). They serve as summaries of the standards at each EFL and for each of the four domains.

ABE 1: Beginning Literacy

The Mathematical Practices: Students prepared to exit ABE 1

- can decipher a simple problem presented in a context.
- reason about and apply correct units to the results.
- visualize a situation using manipulatives or drawings.
- explain their processes and results using mathematical terms and symbols appropriate for the level.
- recognize errors in the work and reasoning of others.
- are able to strategically select and use appropriate tools to aid in their work, such as pencil/paper, measuring devices, and/or manipulatives.
- can see patterns and structure in sets of numbers and geometric shapes and use those insights to work more efficiently.

Number Sense and Operations: Students prepared to exit ABE 1

- understand whole number place value for tens and ones.
- can use their understanding of place value to compare two-digit numbers.
- can add whole numbers within 100 and explain their reasoning (e.g., using concrete models or drawings and strategies based on place value and/or properties of operations).
- can apply their knowledge of whole number addition and subtraction to represent and solve word problems that call for addition of three whole numbers whose sum is less than 20 by using such problem-solving tools as objects, drawings, and/or simple equations.

Algebraic Thinking: Students prepared to exit ABE 1

- understand and apply properties of operations to addition and subtraction problems.
- understand the relationship between the two operations and
- can determine the unknown number in addition or subtraction equations.

Geometry and Measurement: Students prepared to exit ABE 1

- can analyze and compare 2-dimensional and 3-dimensional shapes based on their attributes, such as
    - shape,
    - size,
    - orientation,
    - number of sides and/or vertices (angles), and
    - lengths of sides.
- can reason with two-dimensional shapes (e.g., right prisms, cones, cylinders) to create composite shapes.
    - can measure the length of an object as a whole number of units, which are not necessarily standard units (e.g., measuring the length of a pencil using a paper clip as the length unit)

Data Analysis: Students prepared to exit ABE 1

- can organize, represent, and interpret simple data sets (e.g., lists of numbers, shapes, or items) using up to three categories.
- can answer basic questions related to the total number of data points in a set and the number of data points in each category.
- can compare the number of data points in the different categories.

ABE 2: Beginning Basic

The Mathematical Practices: Students prepared to exit ABE 2

- are able to decipher two-step problems presented in a context, visualizing a situation using diagrams or sketches, and reasoning about and applying the correct units and the proper degree of precision to the results.
- can explain their processes and results using mathematical terms and symbols appropriate for the level
- recognize errors in the reasoning of others.
- strategically select and use the appropriate tools to aid in their work, such as pencil/paper, measuring devices, manipulatives, and/or calculators.
- are able to see patterns and structure in sets of numbers, including in multiplication or addition tables, and use those insights to work more efficiently.

Number Sense and Operations: Students prepared to exit ABE 2

- understand place value for whole numbers to 1000.
- can use that understanding to read, write, count, compare, and round three-digit whole numbers to the nearest 10 or 100.
- are able to compute fluently with all four operations with whole numbers within 100.
- use place value and properties of operations to explain why addition and subtraction strategies work.
- can demonstrate an understanding of the inverse relationship between multiplication and division.
- can solve one- and two-step word problems involving all four operations within 100 and identify and explain arithmetic patterns.
- have an understanding of fractions, especially unit fractions.
- can represent simple fractions on a number line.
- understand and can explain equivalence of fractions.
- can recognize and generate simple equivalent fractions.
- can compare two fractions with the same numerator or denominator by reasoning about their size.

Algebraic Thinking: Students prepared to exit ABE 2

- apply the properties of operations to multiplication and division of whole numbers.
- understand the relationship between multiplication and division.
- can determine the unknown number in multiplication or division equations.

Geometry and Measurement: Students prepared to exit ABE 2

- are able to reason about geometric shapes and their attributes.
- can demonstrate an understanding that different shapes might share common attributes (e.g., four sides)
- can compare and classify two-dimensional shapes, particularly quadrilaterals.
- are able to partition shapes into parts with equal areas and can express the area of each part as a unit fraction of the whole.
- can use common U.S. Customary and metric units for linear measurements (e.g., inches, feet, centimeters, and meters).
- can solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- understand the concept of area and can relate it to addition and multiplication to solve real-world problems.
- understand and can solve real-world and mathematical problems involving perimeter of polygons.

Data Analysis: Students prepared to exit ABE 2

- are able to draw and interpret simple graphs (e.g., bar graphs, picture graphs, and number line diagrams), including scaled bar and picture graphs.
- can solve one- and two-step problems using scaled bar graphs.
- can generate measurement data by measuring lengths to the nearest half- and quarter-inch and display that data by making a line plot marked off in appropriate units.

ABE 3: Low Intermediate

***The Mathematical Practices*:** Students prepared to exit ABE 3

- are able to decipher multi-step problems presented in a context.
- can reason about and apply the correct units and the proper degree of precision to the results.
- can visualize a situation using diagrams or sketches.
- can see multiple strategies for solving a problem.
- can explain their processes and results.
- can recognize errors in the work and reasoning of others.
- can express themselves using mathematical terms and notation appropriate for the level.
- can strategically select and use tools to aid in their work, such as pencil/paper, measuring devices, and/or technology.
- are able to see patterns and structure in sets of numbers and geometric shapes and use those insights to work more efficiently.

Number Sense and Operations: Students prepared to exit ABE 3

- understand place value for both multi-digit whole numbers and decimals to thousandths.
- use their understanding to read, write, compare, and round decimals.
- are able to use their place value understanding and properties of operations to fluently perform operations with multi-digit whole numbers and decimals.
- can find common factors, common multiples, and understand fraction concepts, including fraction equivalence and comparison.
- can add, subtract, multiply, and divide with fractions and mixed numbers.
- are able to solve multi-step word problems posed with whole numbers and fractions, using the four operations.
- understand of ratio concepts and can use ration language to describe a relationship between two quantities, including the concept of a unit rate associated with a ratio.

Algebra: Students prepared to exit ABE 3

- are able to apply and extend their understanding of arithmetic to algebraic expressions, using a symbol to represent an unknown value.
- can write, evaluate, and interpret expressions and equations, including expressions that arise from formulas used in real-world problems.

- can solve real-world and mathematical problems by writing and solving simple one-variable equations and write a simple inequality that represents a constraint or condition in a real-world or mathematical problem.
- can represent and analyze quantitative relationships between dependent and independent variables.

Geometry and Measurement: Students prepared to exit ABE 3

- have a basic understanding of the coordinate plane and can plot points (i.e., ordered pairs) and place polygons in the coordinate plane to solve real-world and mathematical problems.
- can classify two-dimensional shapes and use formulas to determine the area of two-dimensional shapes such as triangles and quadrilaterals.
- can determine the surface area of three-dimensional shapes composed of rectangles and triangles, and find the volume of right rectangular prisms.
- are able to convert like measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m) and use these conversions to solve multi-step, real-world problems.
- are also able to solve measurement word problems (such as those that involve area, perimeter, distance, time intervals, liquid volumes, mass, and money) that involve simple fractions or decimals.

Data Analysis and Statistics: Students prepared to exit ABE 3

- have a basic conceptual understanding of statistical variability, including such concepts as center, spread, and the overall shape of a distribution of data.
- can present data using displays such as dot plots, histograms, and box plots.

ABE 4: High Intermediate

***The Mathematical Practices*:** Students prepared to exit ABE 4

- are able to think critically, determine an efficient strategy (from among multiple possible strategies) for solving a multi-step problem, and persevere in solving challenging problems.
- can express themselves using the mathematical terms and notation appropriate to the level.
- are able to defend their findings and critique the reasoning of others.
- are accurate in their calculations and use estimation strategies to assess the reasonableness of their results.
- can create algebraic and geometric models and use them to answer questions and solve problems.
- can strategically select and use tools to aid in their work, such as pencil/paper, measuring devices, calculators, and/or spreadsheets.
- are able to see patterns and structure in number sets, data, expressions and equations, and geometric figures.

Number Sense and Operations: Students prepared to exit ABE 4

- understand the rational number system, including how rational numbers can be represented on a number line and pairs of rational numbers can be represented on a coordinate plane.
- can apply the concept of absolute value to find horizontal and vertical distances.
- are able to apply the properties of integer exponents and evaluate, estimate, and compare simple square roots and cube roots.
- understand ratio, rate, and percent concepts, as well as proportional relationships.

Algebraic Thinking: Students prepared to exit ABE 4

- understand connections between proportional relationships, lines, and linear equations.
- understand numerical and algebraic expressions and equations.
- are able to use them to solve real-world and mathematical problems.
- are able to analyze and solve linear equations and pairs of simultaneous linear equations.
- are able to define, interpret, and compare linear functions.

Geometry and Measurement: Students prepared to exit ABE 4

- can solve real-world and mathematical problems that involve angle measure, circumference, and area of 2-dimensional figures.
- are able to solve problems involving scale drawings of 2-dimensional geometric figures.
- understand the concepts of congruence and similarity with respect to 2-dimensional figures.
- understand the Pythagorean theorem and can apply it to determine missing lengths in right triangles.

Data Analysis and Statistics: Students prepared to exit ABE4

- can summarize and describe numerical data sets in relation to their context, including determining measures of center and variability and describing patterns and/or striking deviations from patterns.
- understand and can apply the concept of chance, or probability.
- are able to use scatter plots for bivariate measurement data to describe patterns of association between two quantities (such as clustering, outliers, positive or negative association, linear or non-linear association).

ABE 5: Low Adult Secondary

The Mathematical Practices: Students prepared to exit ABE 5

- are able to think critically,
- determine an efficient strategy (from among multiple possible strategies) for solving a multi-step problem, and
- persevere in solving challenging problems.
- can reason quantitatively, including using units as a way to solve problems.

- are able to defend their findings and critique the reasoning of others.
- are accurate in their calculations and use estimation strategies to assess the reasonableness of their results.
- can create algebraic and geometric models and use them to answer questions and solve problems.
- can strategically select and use tools to aid in their work, such as graphing calculators, spreadsheets, and/or computer software.
- are able to make generalizations based on patterns and structure they discover in number sets, data, expressions and equations, and geometric figures and use these insights to work more efficiently.

Number Sense and Operations: Students prepared to exit ABE 5
- can reason about and solve real-world and mathematical problems that involve the four operations with rational numbers.
- can apply the concept of absolute value to demonstrate on a number line their understanding of addition and subtraction with negative and positive rational numbers.
- can apply ratio and percent concepts, including using rates and proportional relationships to solve multi-step real-world and mathematical problems.

Algebraic Thinking: Students prepared to exit ABE 5
- are able to use algebraic and graphical representations to solve real-world and mathematical problems, involving linear equations, inequalities, and pairs of simultaneous linear equations.
- are able to use linear functions to describe, analyze, and model linear relationships between quantities.

Geometry and Measurement: Students prepared to exit ABE 5
- can solve real-world and mathematical problems that involve volume and surface area of 3-dimensional geometric figures.
- can use informal arguments to establish facts about various angle relationships such as the relationships between angles created when parallel lines are cut by a transversal.
- apply the Pythagorean theorem to determine lengths in real-world contexts and distances in the coordinate plane.

Data Analysis and Statistics: Students prepared to exit ABE 5

- can use random sampling to draw inferences about a population.
- are able to draw informal comparative inferences about two populations using measures of center and measures of variability for numerical data from random samples.
- can develop, use, and evaluate probability models.
- are able to use scatter plots for bivariate measurement data to interpret patterns of association between two quantities (such as clustering, outliers, positive or negative association, linear or non-linear association) and a 2-way table to summarize and interpret bivariate categorical data.

ABE 6: High Adult Secondary

***The Mathematical Practices*:** Students prepared to exit ABE 6

- are able to think critically,
- make assumptions based on a situation,
- select an efficient strategy from multiple possible problem- solving strategies,
- plan a solution pathway, and
- make adjustments as needed when solving problems.
- persevere in solving challenging problems, including considering analogous, simpler problems as a way to solving a more complex one.
- can reason quantitatively, including through the use of units, and
- can express themselves using the precise definitions and mathematical terms and notation appropriate to the level.
- are accurate in their calculations,
- use an appropriate level of precision in finding solutions and reporting results, and
- use estimation strategies to assess the reasonableness of their results.
- are able to make conjectures,
- use logic to defend their conclusions, and
- can detect faulty thinking and errors caused by improper use of technology.
- can create algebraic and geometric models and use them to
    - answer questions,
    - interpret data,
    - make predictions, and
    - solve problems.
- can create algebraic and geometric models and use them to
    - answer questions,
    - interpret data,

    - make predictions, and
    - solve problems.
- can strategically select and use tools, such as
    - measuring devices,
    - calculators,
    - spreadsheets, and/or
    - computer software, to aid in their work.
    - are able to see patterns and structure in calculations, expressions, and equations and make connections to algebraic generalizations, which they use to work more efficiently.

Number Sense and Operations: Students prepared to exit ABE 6

- have extended their number sense to include irrational numbers, radicals, and rational exponents.
- understand and use the set of real numbers.
- are able to assess the reasonableness of calculation results based on the limitations of technology or given units and quantities and give results with appropriate degree of precision.

Algebraic Thinking: Students prepared to exit ABE 6

- understand the structure of expressions and can use that structure to rewrite linear, exponential, and quadratic expressions.
- can add, subtract, and multiply polynomials that involve linear and/or quadratic expressions.
- are also able to create linear equations and inequalities and quadratic and simple exponential equations to represent relationships between quantities and can represent constraints by linear equations or inequalities, or by systems of linear equations and/or inequalities.
- can interpret the structure of polynomial and rational expressions and use that structure to identify ways to rewrite and operate accurately with them.
- can add, subtract, and multiply polynomials that extend beyond quadratics.
- are able to rearrange formulas to highlight a quantity of interest, for example rearranging Ohm's law, V = IR, to highlight resistance R.
- are also able to create equations and inequalities representing relationships between quantities, including those that extend beyond equations or inequalities arising from linear, quadratic, and simple exponential functions to include those arising from simple rational functions.
- are able to use these equations/inequalities to solve problems both algebraically and graphically.
- can solve linear equations and inequalities; systems of linear equations; quadratic, simple rational, and radical equations in one variable; and recognize how and when extraneous solutions may arise.

Students prepared to exit ABE 6 also

- have a basic understanding of functions.
- can use function notation properly.
- use such notation to write a function describing a relationship between two quantities.
- are able to evaluate functions for inputs in their domains and interpret linear, quadratic, and exponential functions that arise in applications in terms of the context.
- are able to construct, graph, compare, and interpret functions (including, but not limited to, linear, quadratic, and exponential).
- can sketch graphs given a verbal description of the relationship and identify and interpret key features of the graphs of functions that arise in applications in a context.
- are able to select or define a function that appropriately models a relationship and to compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal description).

Geometry and Measurement: Students prepared to exit ABE 6

- can solve problems involving similarity and congruence criteria for triangles.
- can use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- can apply the concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTU's per cubic foot).

Data Analysis and Statistics: Students prepared to exit ABE 6

- can summarize, represent, and interpret data based on two categorical and quantitative variables, including by using frequency tables.
- can compare data sets by looking at commonalities and differences in shape, center, and spread.
- can recognize possible associations and trends in data, in particular in linear models, and distinguish between correlation and causation.
- interpret one- and two-variable data, including those with linear and non-linear relationships.
- interpret the slope (rate of change) and intercept (constant term) for a line of best fit and in the context of the data. They understand and account for extreme points of data in their analysis and interpret relative frequencies (joint, marginal and conditional).

Appendix B: Key Considerations for Standards Implementation in ABE 1 – 4

Adapted from Arizona [K-12] Mathematics Standards Introduction (Arizona Department of Education, High Academic Standards for Students, Dec. 2016)

Addition/Subtraction and Multiplication/Division Problem Types or Situations

There are important distinctions among different types of addition/subtraction and multiplication/division problems that are reflected in the ways that students think about and solve them.[12]

Table 1 and Table 2 describe different problem types that provide structures for selecting problems for instruction and interpreting how students solve them. This is a critical consideration for standards implementation. When planning instruction, educators must provide all students with the opportunity to learn and experience all different problem types associated with a given standard. Without the opportunity to learn and experience different problem types, students cannot truly master and apply the educational functioning level (EFL) standards in future mathematical tasks and experiences.

Table 1: Addition and Subtraction Situations

Table 1 provides support to clarify the varied problem structures necessary to build student conceptual understanding of addition and subtraction, focusing on developing student flexibility. To fully implement the standards, students must solve problems from all problem subcategories relevant to the EFL.

Table 2: Multiplication and Division Situations

Table 2 provides support to clarify the varied problem structures necessary to build student conceptual understanding of multiplication and division, focusing on developing student flexibility. To fully implement the standards, students must solve problems from all problem subcategories relevant to the EFL.

[12] Carpenter, T., Fennema, E., Franke, M., Levi, L., Empson, S. (1999). Children's Mathematics Cognitively Guided Instruction

Table 1. Common Addition and Subtraction Problem Types/Situations[13]

	Result Unknown	**Change Unknown**	**Start Unknown**
Add to	Two apples were on the table. I put three more apples with them. How many apples are on the table now? • 2 + 3 = ?	Two apples were on the table. I put some more apples with them. Then there were five apples. How many apples did I put with the first two? • 2 + ? = 5	Some apples were on the table. I put three more apples with them. Then there were five apples. How many apples were already on the table? • ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? • 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? • 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? • ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown****
Put together / Take apart*	Three red apples and two green apples are on the table. How many apples are on the table? • 3 + 2 = ?	Five apples are on the table. Three are red, and the rest are green. How many apples are green? • 3 + ? = 5, 5 – 3 = ?	I have five apples. How many can I eat myself, and how many can I give to someone else? 5 = 0 + 5 5 = 5 + 0 5 = 3 + 2 5 = 1 + 4 5 = 4 + 1 5 = 2 + 3
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	**"How many more?" version**: Juan has two apples. Jenna has five apples. How many more apples does Jenna have than Juan? • 2 + ? = 5 **"How many fewer?" version**: Juan has two apples. Jenna has five apples. How many fewer apples does Juan have than Jenna? • 5 – 2 = ?	**Version with "more"**: Jenna has three more apples than Juan. Juan has two apples. How many apples does Jenna have? • 2 + 3 = ? **Version with "fewer"**: Juan has 3 fewer apples than Jenna. Juan has two apples. How many apples does Jenna have? • 3 + 2 = ?	**Version with "more"**: Jenna has three more apples than Juan. Jenna has five apples. How many apples does Juan have? • 5 – 3 = ? **Version with "fewer"**: Juan has 3 fewer apples than Jenna. Jenna has five apples. How many apples does Juan have? • ? + 3 = 5

* These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help students understand that the = sign does not always mean "makes" or "results in." However, it does always mean "is the same as."

** Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

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[13] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33)

Table 2. Common Multiplication and Division Problem Types/Situations

	Unknown Product	Group Size Unknown "How many in each group?" Division	Number of Groups Unknown "How many groups?" Division
	3 • 6 = ?	**3 • ? = 18 and 18 ÷ 3 = ?**	**? • 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? ***Measurement example:*** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? ***Measurement example:*** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? ***Measurement example:*** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays* Area**	There are 3 rows of apples with 6 apples in each row. How many apples are there? ***Area example:*** What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? ***Area example:*** A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? ***Area example:*** A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A pair of socks costs $3. A t-shirt costs 3 times as much as the socks. How much does the t-shirt cost? ***Measurement Example:*** A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A t-shirt costs $9, and that is 3 times as much as a pair of socks costs. How much does a pair of socks cost? ***Measurement example:*** A rubber band is stretched to be 18 cm long, and that is 3 times as long as it was at first. How long was the rubber band at first?	A t-shirt costs $9, and a pair of socks costs $3. How many times as much does the t-shirt cost as the socks? ***Measurement example:*** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$A \cdot b = ?$	$a \cdot ? = p$ and $p \div a = ?$	$? \cdot b = p$ and $p \div b = ?$

Appendix C: Properties Tables

TABLE 1. The properties of operations

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $–a$ so that $a + (–a) = (–a) + a = 0$.
Associative property of multiplication	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Commutative property of multiplication	$a \cdot b = b \cdot a$
Multiplicative identity property of 1	$a x 1 = 1 x a = a$
Existence of multiplicative inverses	For every $a \neq 0$, there exists $1/a$ so that a x $1/a = 1/a$ x $a = 1$.
Distributive property of Multiplication over addition	$a \cdot (b + c) = a \cdot b + a \cdot c$

TABLE 2. The properties of equality

Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a=a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a\ c = b\ c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \cdot c = b \cdot c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

TABLE 3. The properties of inequality

Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, or a > b
If a > b and b > c, then a > c.
If a > b, then b < a.
If a > b, then –a < –b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b ⋅ c
If a > b and c < 0, then a ⋅ c < b ⋅ c
If a > b and c > 0, then a + c > b + c
If a > b and c < 0, then a + c < b + c